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Studying Rotating Objects in Hyperbolic Symmetry under the *1+3-* Formalism

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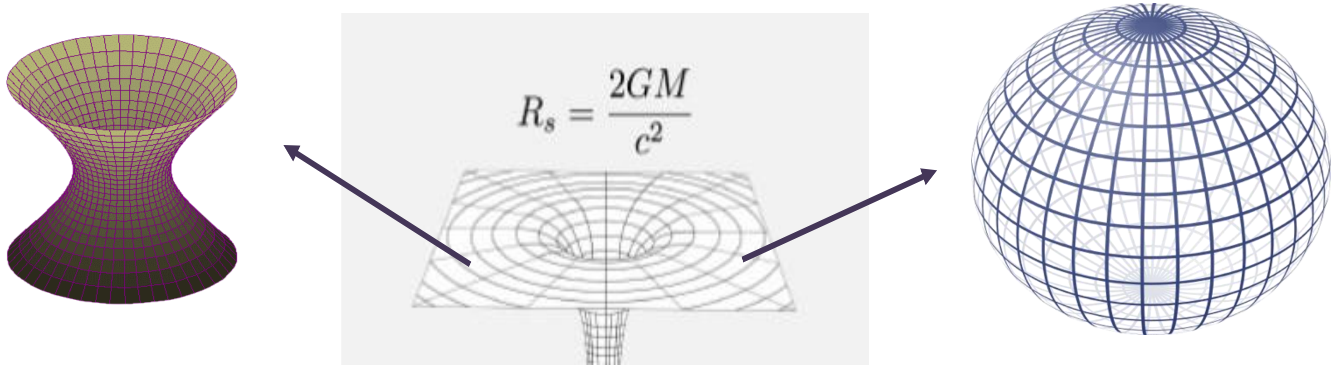
Overview of Findings

Through the 1+3 formalism we obtained

1. **A strategy to find vacuum new solutions based on the definition of functions related to the physics of spacetime's configuration.**

3. **The Kerr metric and the hyperbolic version, are the only metrics admitting a Killing tensor with suitable asymptotic conditions for the Killing equation.**

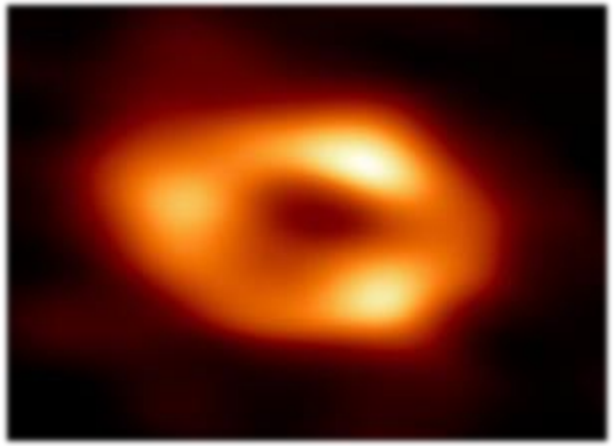
2. **The version for the Kerr metric in hyperbolic symmetry r <2m.**



L. Herrera , and Louis Witten, 2018

Currents observations and some open questions.

First Black hole image M87, 2019



SgA* image, 2022

From the imagen of Black hole M87 or SgA*

- **Can we establish a relationship between the dynamic around, and properties of the spacetime?**

- **Which aspects of the spacetime play an important role in the image?**

About the imagen of Black hole M87 or SgA*

- **Can we say something about the photon orbits under this formalism?**

Black Hole Images as Tests of General Relativity: Efects of Spacetime Geometry. Z.Younsi, D.Psaltis, and F.Ozel Astrophysical Journal 2023

The strategy of the *1+3 formalism*

- The strategy of the formalism is, to compile two independent set of equation in terms of scalars function containing the same information than the Einstein Equations.

  - Orthogonal unitary tetrad

$$e_{\alpha}^{(0)} = V_{\alpha}, e_{\alpha}^{(1)} = K_{\alpha}, e_{\alpha}^{(2)} = L_{\alpha} \text{ and } e_{\alpha}^{(3)} = S_{\alpha}$$

  - The set of equations emerges from

- **Projection of the Riemann tensor along the tetrads**

$$2V_{\alpha;[\beta;\gamma]} = R_{\delta\alpha\beta\gamma}V^{\delta},\ 2K_{\alpha;[\beta;\gamma]} = R_{\delta\alpha\beta\gamma}K^{\delta} \quad 2L_{\alpha;[\beta;\gamma]} = R_{\delta\alpha\beta\gamma}L^{\delta}, \text{ and } 2S_{\alpha;[\beta;\gamma]} = R_{\delta\alpha\beta\gamma}S^{\delta}$$

- **Bianchi identities**

$$R_{\alpha\beta[\gamma\delta;\mu]} = R_{\alpha\beta\gamma\delta;\mu} + R_{\alpha\beta\mu\gamma;\delta} + R_{\alpha\beta\delta\mu;\gamma} = 0$$

- ***Dynamical laws of superenergy in General Relativity***
García-Parrado, Gómez-Lobo, Class.Quant.Grav 2008
- ***An equivalent system of Einstein Equations***, *J. Ospino, J. L. Hernández-Pastora, L. A. Núñez*

The vacuum equations

$$a_1^\dagger + a_2^* + a_1(a_1 + J_2 + J_6) + a_2(a_2 + J_9 - J_1) + 2(\Omega_2^2 + \Omega_3^2) = 0,$$

$$\Omega_2^\dagger + \Omega_2(2a_1 + J_2) + \Omega_3^* + \Omega_3(2a_2 - J_1) = 0,$$

$$J_1^* - (a_1 + J_2 + J_6)^\dagger + J_1(a_2 + J_9 - J_1) - a_1^2 - J_2^2 - J_6^2 + 2\Omega_2^2 = 0,$$

$$(a_1 + J_6)^* + a_2(a_1 - J_2) + J_9(J_6 - J_2) - 2\Omega_2\Omega_3 = 0,$$

$$(a_2 + J_9)^\dagger + a_1(a_2 + J_1) + J_1 J_6 + J_6 J_9 - 2\Omega_2\Omega_3 = 0,$$

$$J_2^\dagger + (a_2 - J_1 + J_9)^* + a_1 J_2 + J_2 J_6 + a_2^2 + J_1^2 + J_2^2 + J_9^2 - 2\Omega_3^2 = 0,$$

$$J_6^\dagger + J_9^* + J_6(a_1 + J_2 + J_6) + J_9(a_2 - J_1 + J_9) - 2(\Omega_2^2 + \Omega_3^2) = 0,$$

$\dagger$ $*$ defines the directional derivatives along K y L respectively

Main Outcomes

For some specifics functions in the integration of the equations, we have:

- The Kerr solution $r > 2m$

- $X_{\rho\rho} + \frac{1}{\rho} X_\rho + X_{zz} = 0$

The Weyl equation

- The Kerr solution in hyperbolic symmetry $r < 2m$

$$ds^2 = -\left(1 - \frac{2mr}{\Lambda_{KH}}\right) dt^2 - \frac{4mar\sinh^2\theta}{\Lambda_{KH}} dt d\phi + \frac{\Lambda_{KH}}{2mr - r^2 - a^2} dr^2 + \Lambda_{KH} d\theta^2 + \sinh^2\theta \left(r^2 + a^2 - \frac{2ma^2 r \sinh^2\theta}{\Lambda_{KH}}\right) d\phi^2$$

$$\Lambda_{KH} = r^2 + a^2 \cosh^2\theta$$

Some results on Axial-symmetric stationary vacuum solutions
J. Ospino and J.L. Hernández-Pastora, **A.Araujo**, and L.A.Núñez. ***In preparation***

Axially Symmetric Killing Tensor

For the stationary axially symmetric space-time, that admits a Killing tensor $\xi_{\alpha\beta}$ satisfying the Killing equation

$$\xi_{\alpha\beta;\mu} + \xi_{\mu\alpha;\beta} + \xi_{\beta\mu;\alpha} = 0$$

which in terms of the tetrads turns as

$$\xi_{\alpha\beta} = \xi_{00} V_\alpha V_\beta + \xi_{11} K_\alpha K_\beta + \xi_{22} L_\alpha L_\beta + \xi_{33} S_\alpha S_\beta + \xi_{03}(V_\alpha S_\beta + V_\beta S_\alpha)$$

We found, after integrating the equation, and under suitable asymptotic conditions, the only metrics are

- The Kerr solution $r > 2m$

- The hyperbolic version of the Kerr solution solution.

Analyzing photon orbits

The tangent vector and the norm of the vector

All analytic solutions for geodesic motion in axially symmetric space-times
J. Ospino, J. L. Hernández-Pastora,L. A. Núñez , EPJ 2008

$$Z^{\alpha} \equiv \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\lambda} = z_0 V^{\alpha} + z_1 K^{\alpha} + z_2 L^{\alpha} + z_3 S^{\alpha} \qquad \epsilon = Z_{\alpha} Z^{\alpha} = -z_0^2 + z_1^2 + z_2^2 + z_3^2$$

The geodesic equation

$$Z_{\alpha;\beta} Z^{\beta} = 0, \quad \left\{ \begin{array}{l} z_1 z_1^{\dagger} + z_2 z_2^{\dagger} = j_6 z_3^2 + 2 z_0 z_3 \Omega_2 - a_1 z_0^2 \\ z_1 z_1^{\theta} + z_2 z_2^{\theta} = j_9 z_3^2 + 2 z_0 z_3 \Omega_3 - a_2 z_0^2 \end{array} \right.$$

Circular orbits

$$z_1 = z_2 = 0$$

for photon

$$\epsilon = 0 \qquad z_0^2 = z_3^2$$

$$\left\{ \begin{array}{l} a_1 = J_6 + 2\Omega_2 \\ a_2 = J_9 + 2\Omega_3 \end{array} \right.$$

- There are not circular orbits for photon, at least the space-time metric is stationary

Take away

The 1+3 formalism based on tetrads allows to obtain first order differential equations in terms of the scalars.

We provide a strategy to find vacuum new solutions based on the definition of functions related to the physics of spacetime's configuration.

We find a new stationary solution for the inner region of the black hole.

We proved the Kerr metric and the hyperbolic version are the only metric admitting a Killing tensor with suitable asymptotic conditions for the Killing equation.

Even though the **new stationary solution** for the region inside the horizon is desvíated from the classical approach, it represented a good scenario for some astrophysical observations.

Thank you