

# Analysis of anatomical connectivity networks in patients with epilepsy embedded in Euclidean space.

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CNRS



Latin American alliance for  
Capacity building in Advanced physics  
**LA-CoNGA physics**



Cofinanciado por el  
programa Erasmus+  
de la Unión Europea





Data

Diffusion Map

Procrustes

Distances

Nodes

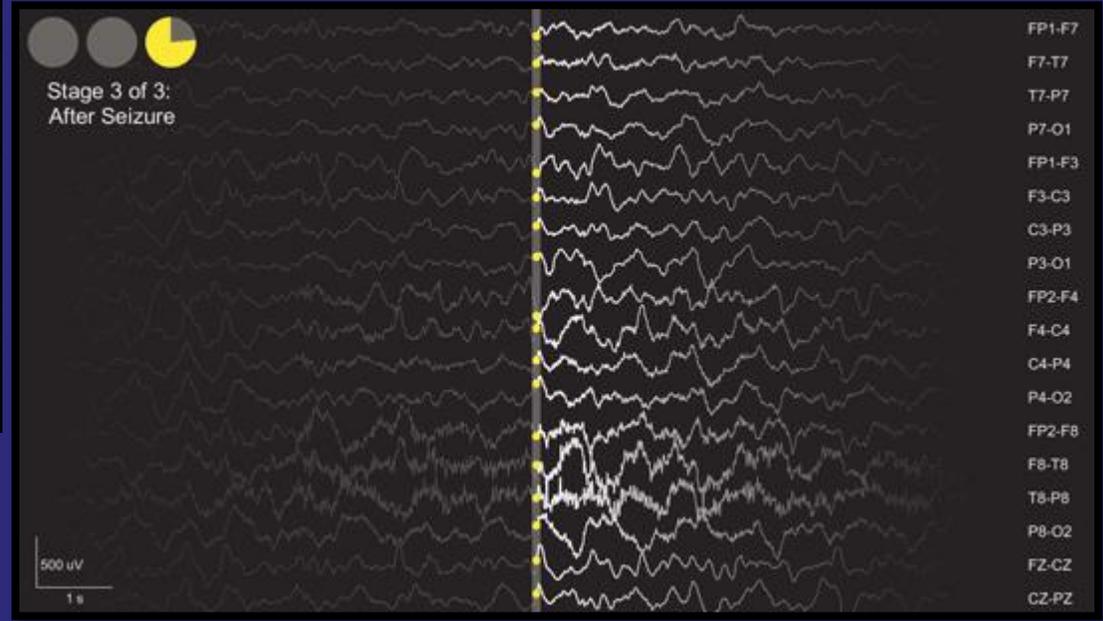
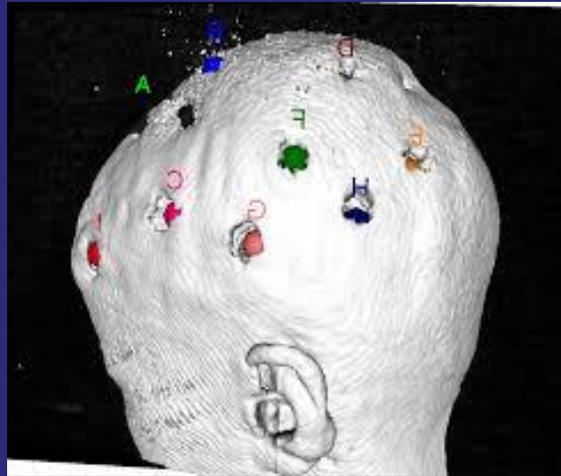




# Data



## intracranial electroencephalography (iEEG)



FP1-F7	
F7-T7	
T7-P7	Delta < 4 Hz
P7-O1	
FP1-F3	Theta 4 – 7 Hz
F3-C3	
C3-P3	Alpha 8 – 12 Hz
P3-O1	
FP2-F4	Beta 13 – 30 Hz
F4-C4	
C4-P4	
P4-O2	
FP2-F8	Low Gamma
F8-T8	
T8-P8	
P8-O2	
FZ-CZ	High Gamma > 30 Hz
CZ-PZ	



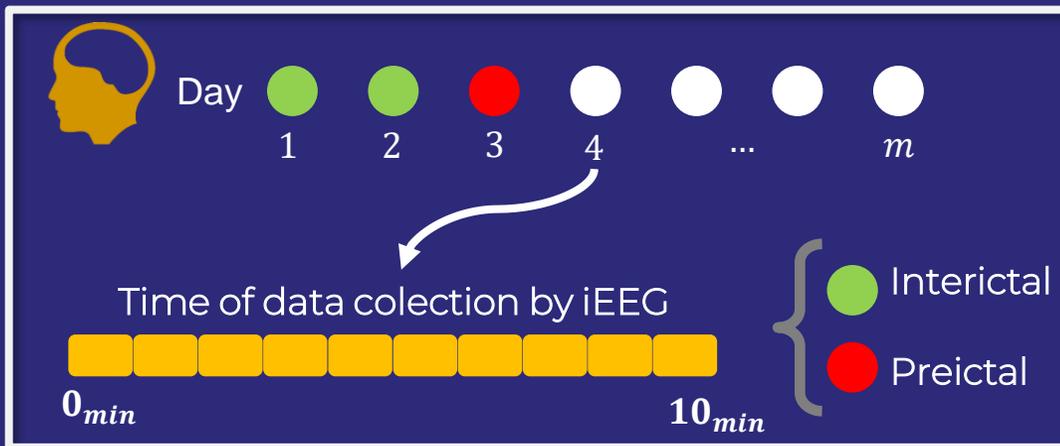
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Procrustes

Distances

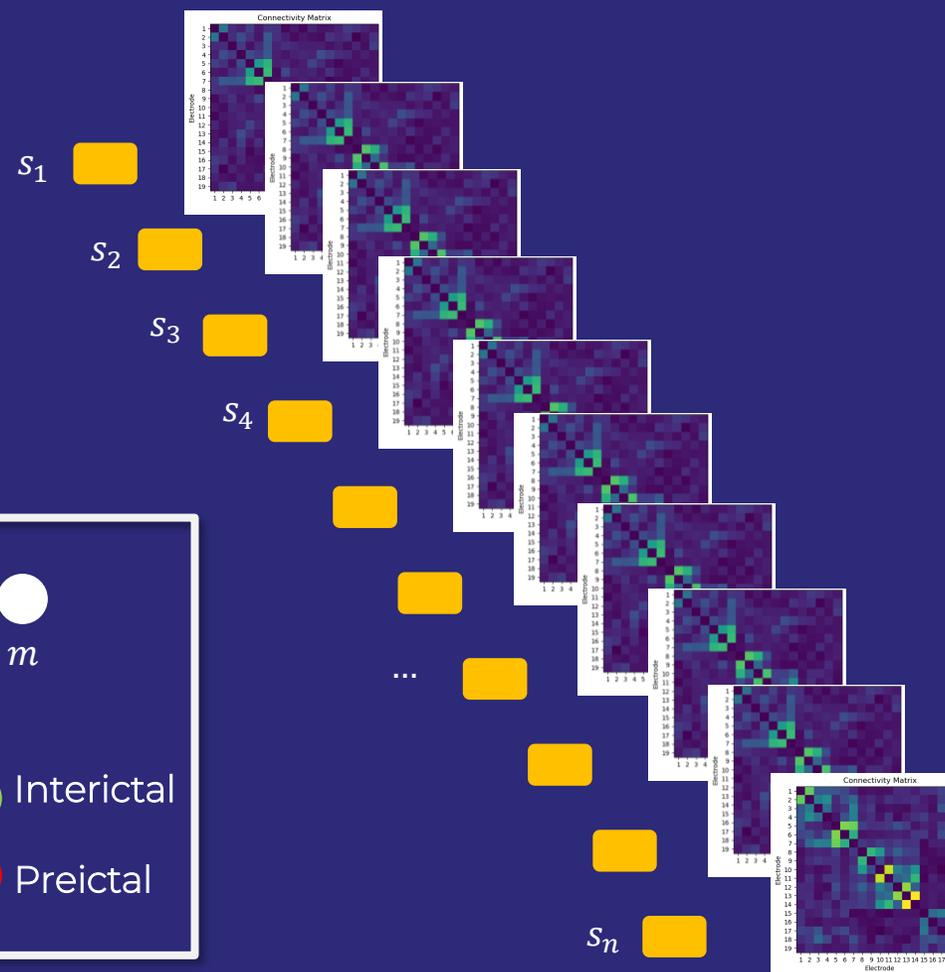
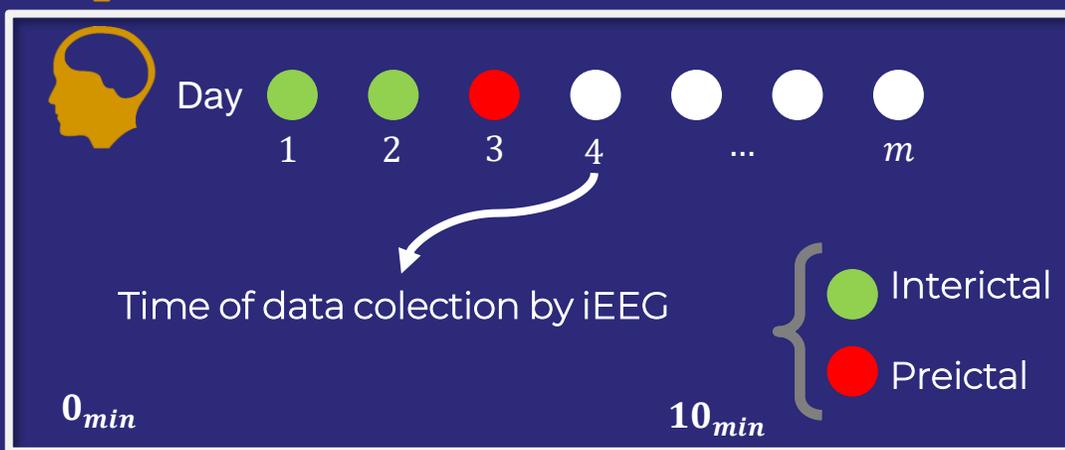
Nodes

# Data





# Data





Data

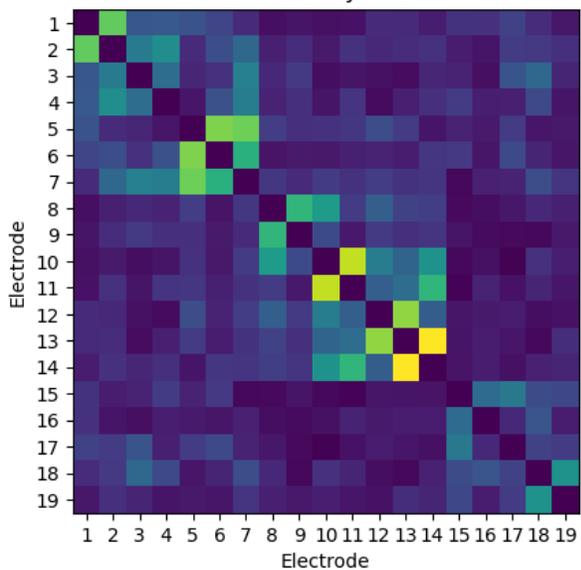
Diffusion Map

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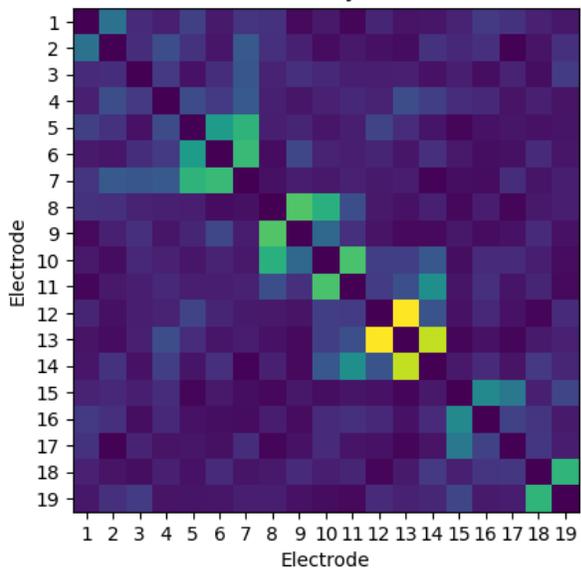
Connectivity Matrix



INTERICTAL



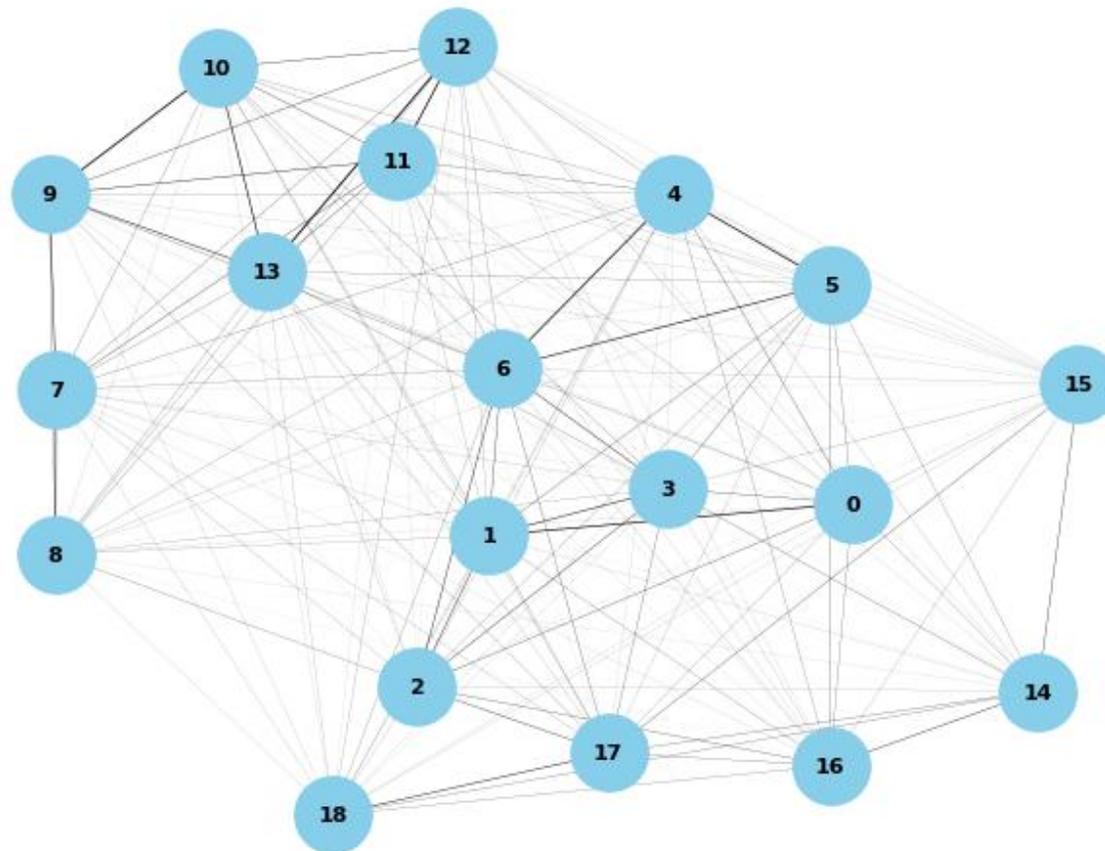
Connectivity Matrix



PREICTAL



## Creation of graphs





Data

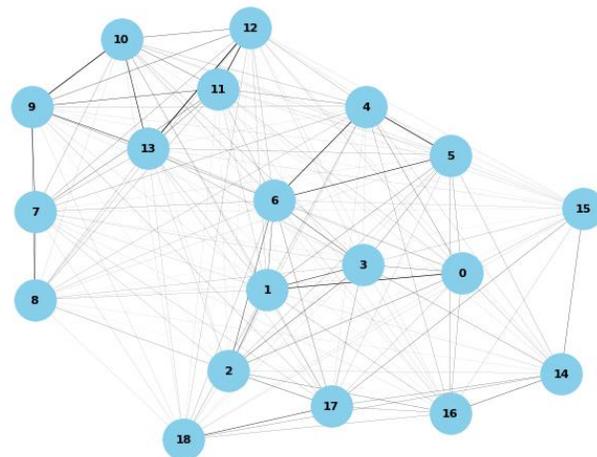
Diffusion Map

Procrustes

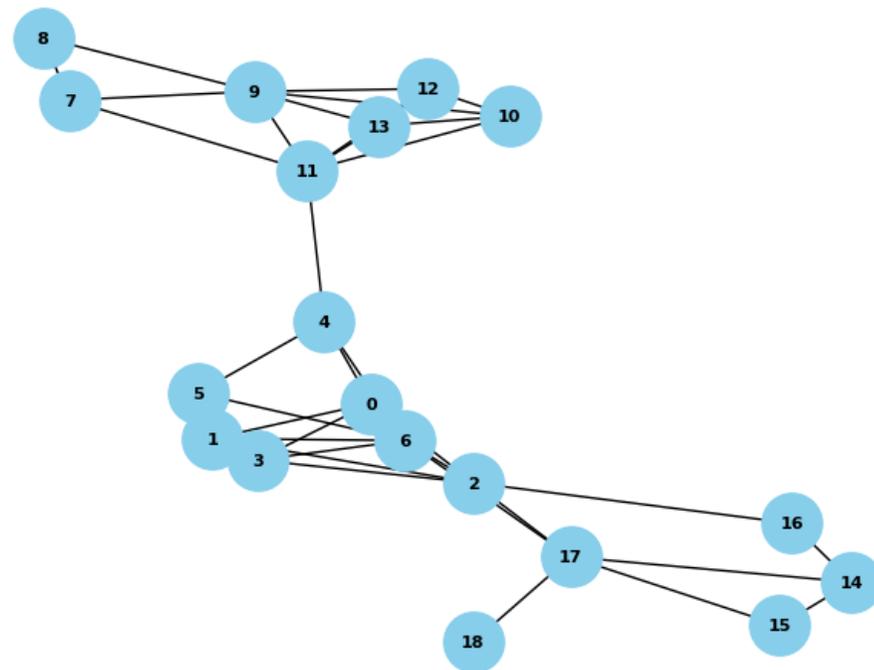
Distances

Nodes

## Creation of graphs



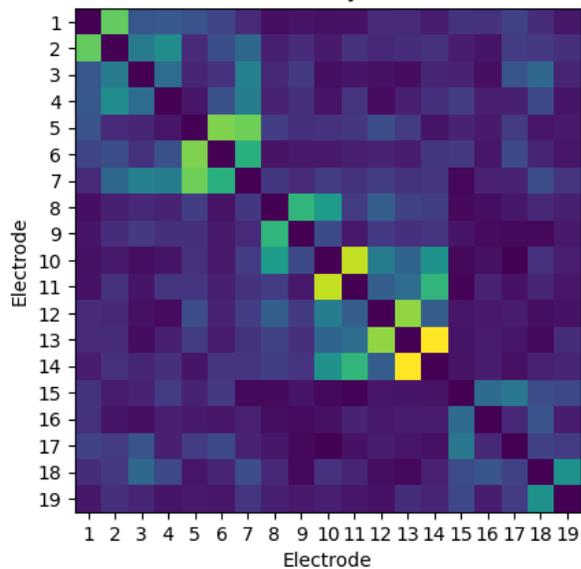
and filtering by spanning tree with mean degree 3



INTERICTAL



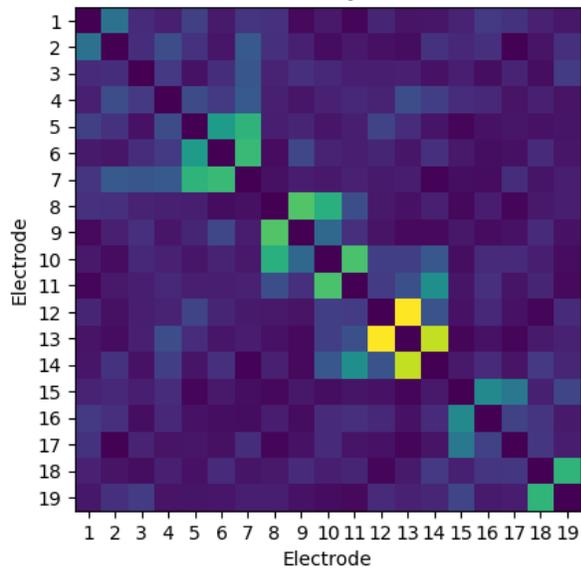
Connectivity Matrix



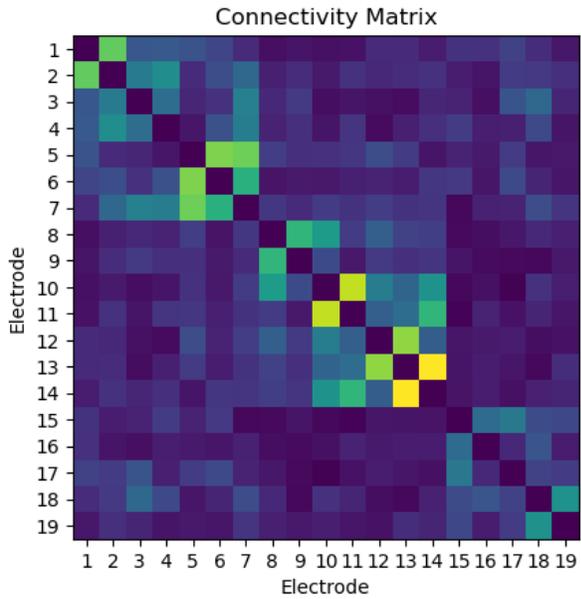
PREICTAL



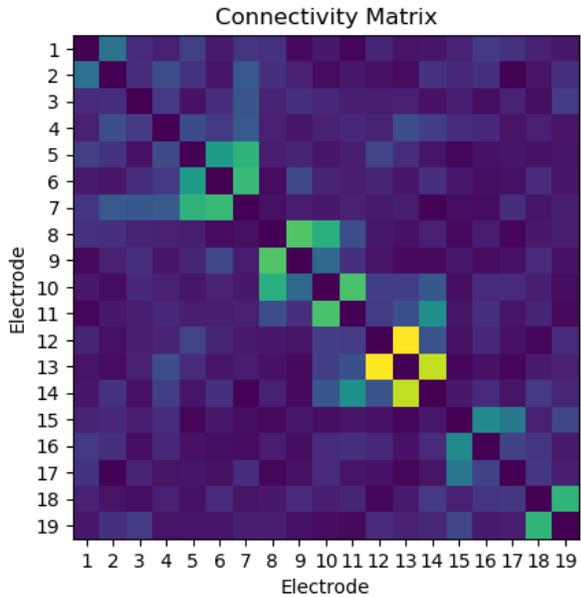
Connectivity Matrix



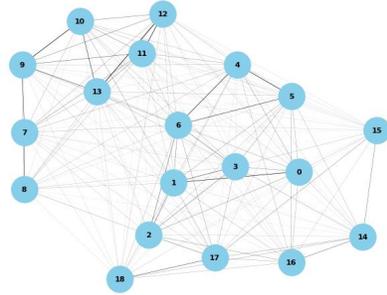
INTERICTAL



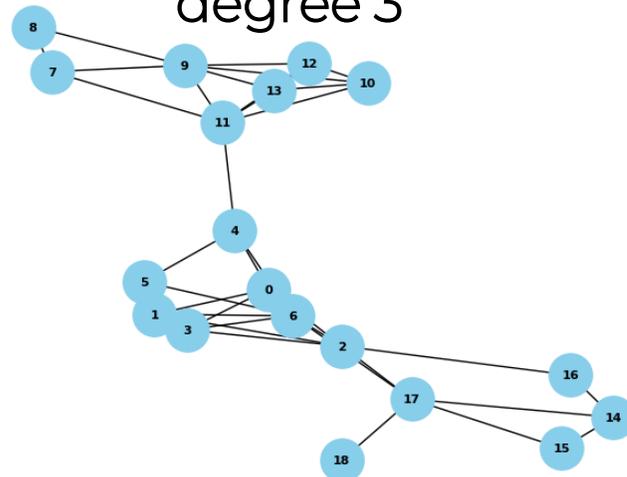
PREICTAL



## Creation of graphs

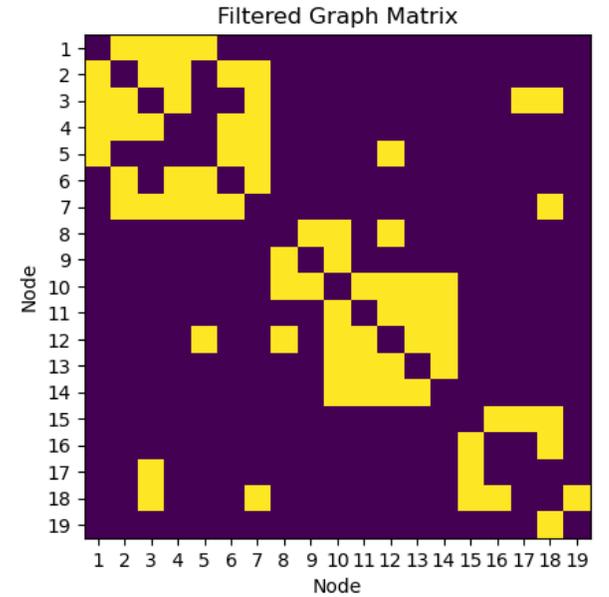


and filtering by  
spanning tree with mean  
degree 3

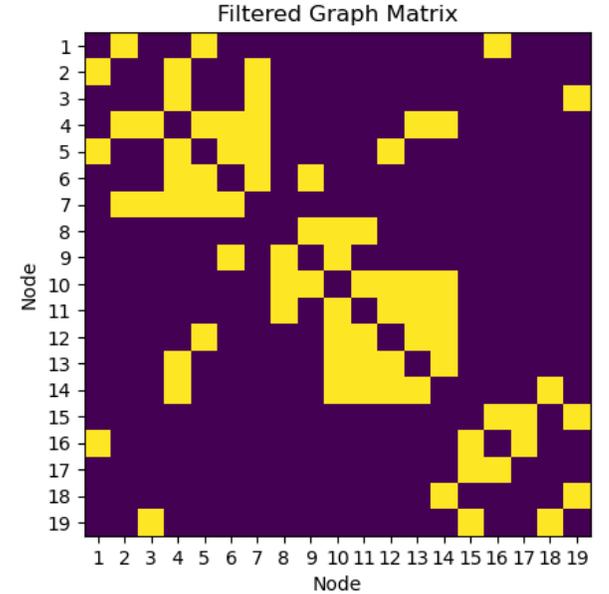


and Binarization

INTERICTAL



PREICTAL



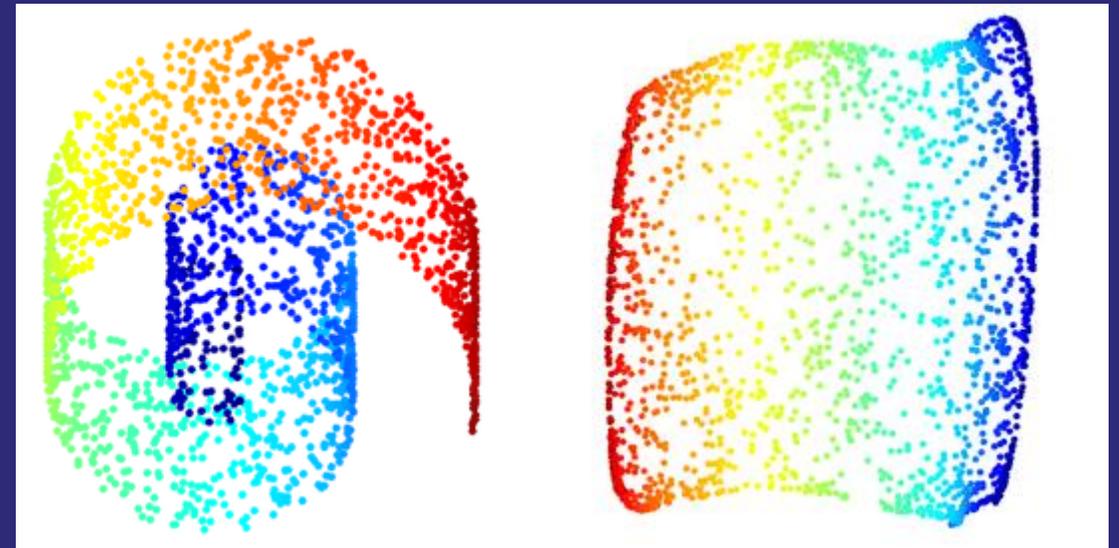


# Diffusion Map

Nonlinear dimensionality reduction technique of a data set in Euclidean space whose coordinates can be computed from the eigenvectors and eigenvalues of a diffusion operator on the data.

This reduction technique works by using the degree of the graph to find the probability of transition between all the data.

The eigenvalues and eigenvectors of the transition matrix provide the representation of the initial graph as a cloud of points in a lower dimensional space.



Shan, S., & Daubechies, I. (2022). Diffusion maps: Using the semigroup property for parameter tuning



Data

Diffusion Map

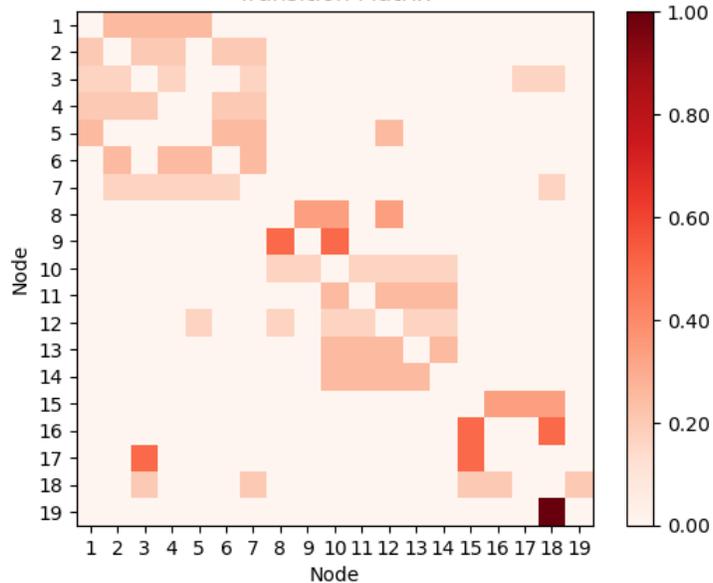
Procrustes

Distances

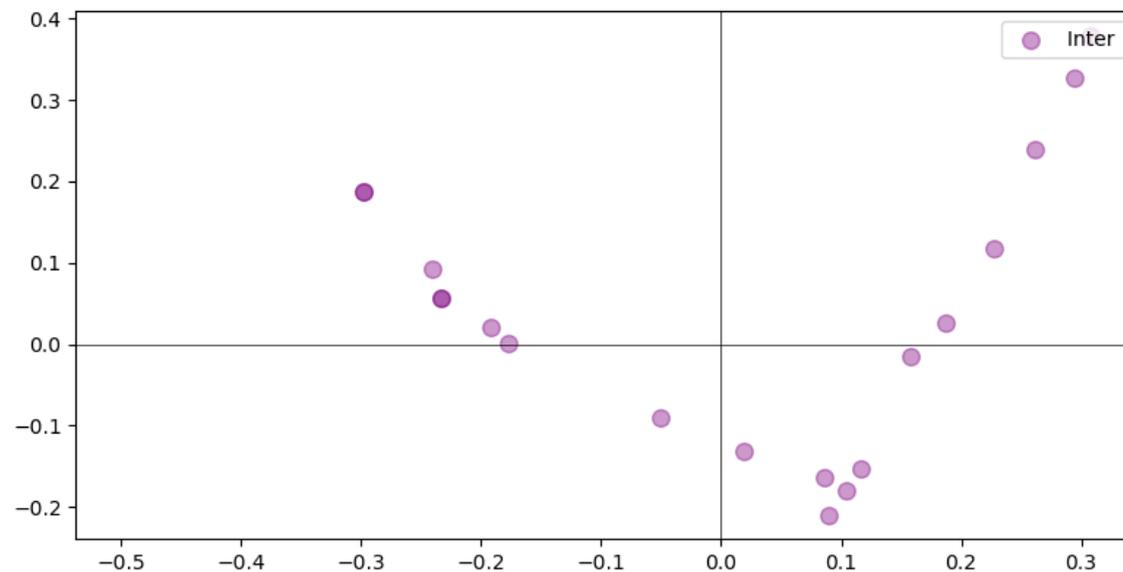
Nodes



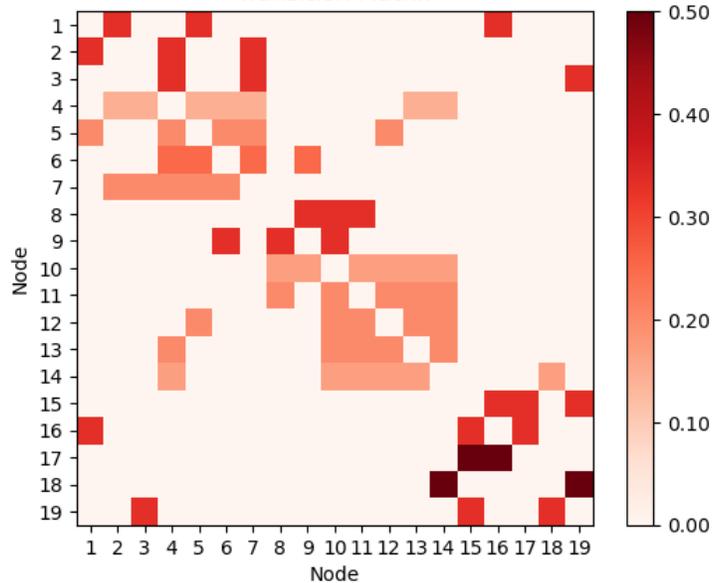
Transition Matrix



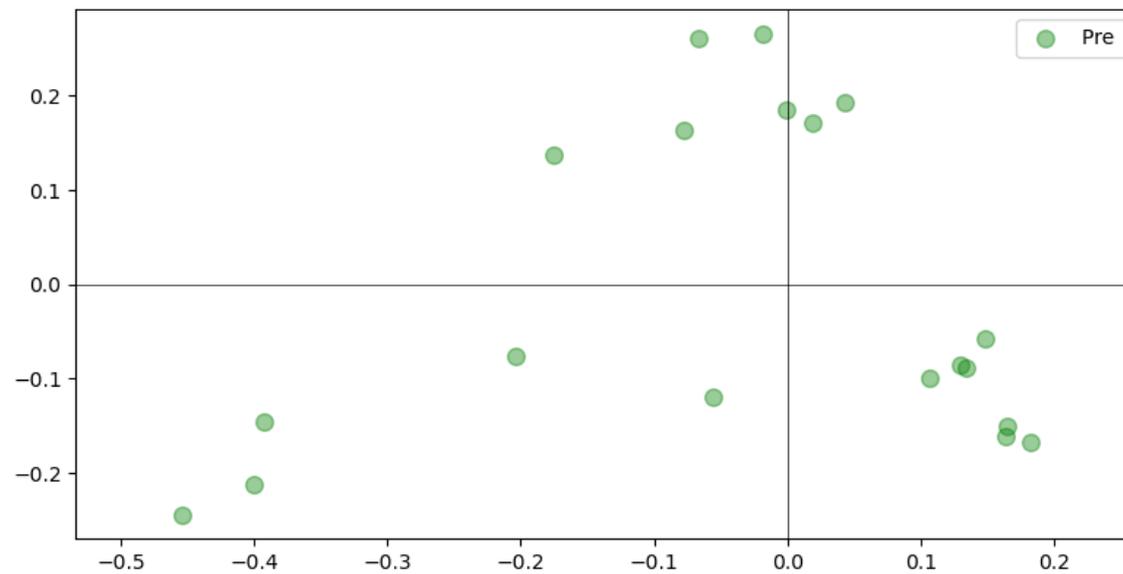
2D Spatial Distribution of the Embedding in Euclidean Space

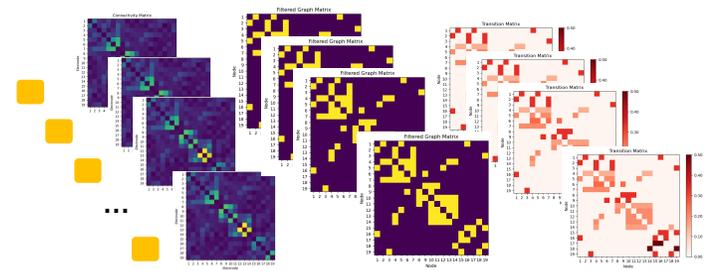
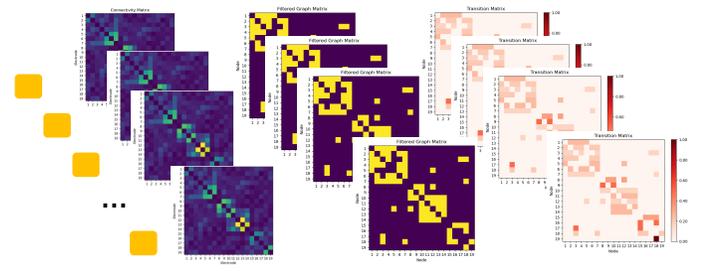


Transition Matrix

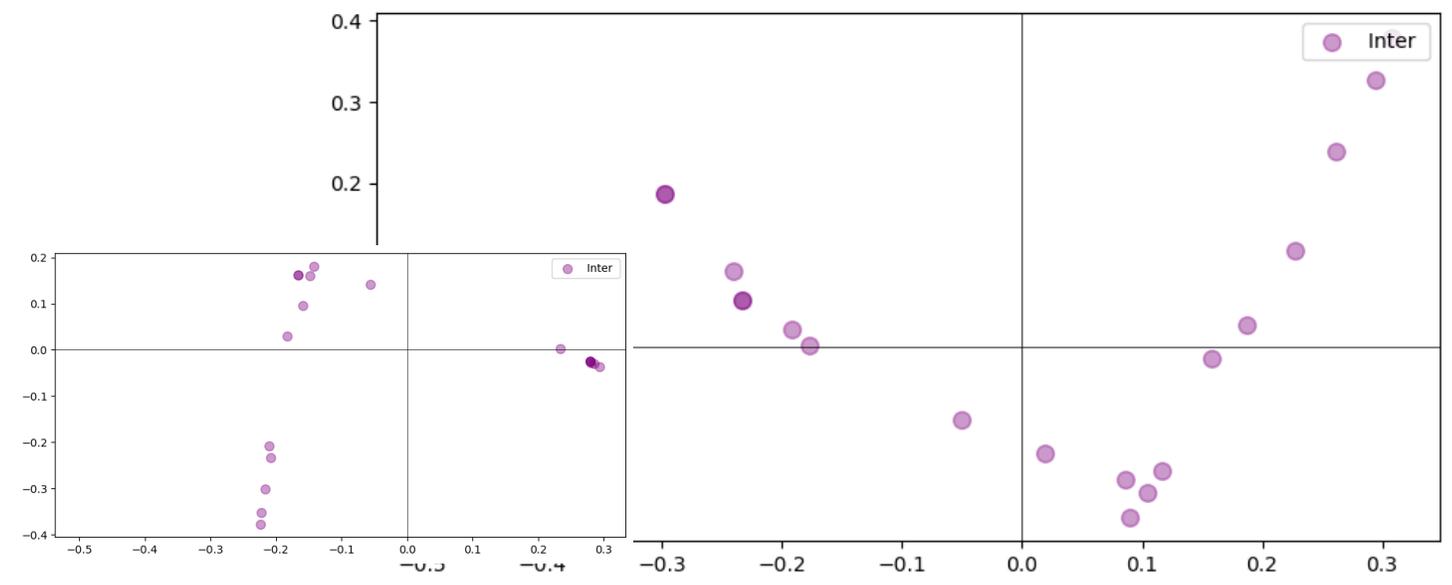


2D Spatial Distribution of the Embedding in Euclidean Space

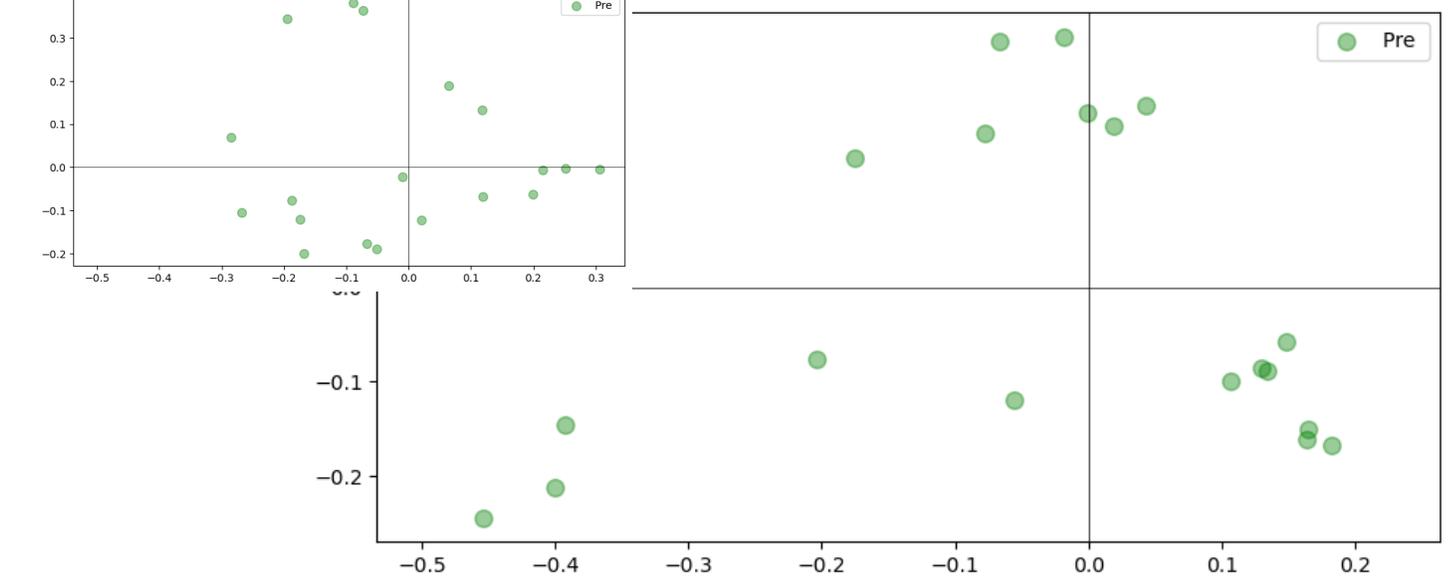




2D Spatial Distribution of the Embedding in Euclidean Space

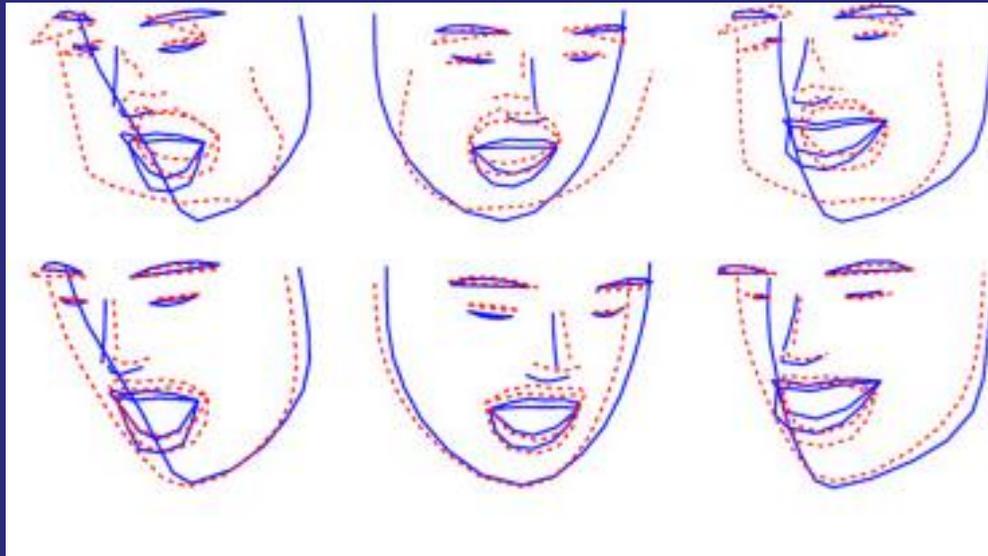


2D Spatial Distribution of the Embedding in Euclidean Space





# Procrustes



Generalized Procrustes Analysis (GPA) is a multivariate exploratory technique that involves transformations (i.e., translation, rotation, reflection and isotropic rescaling) of individual data matrices to provide optimal comparability.



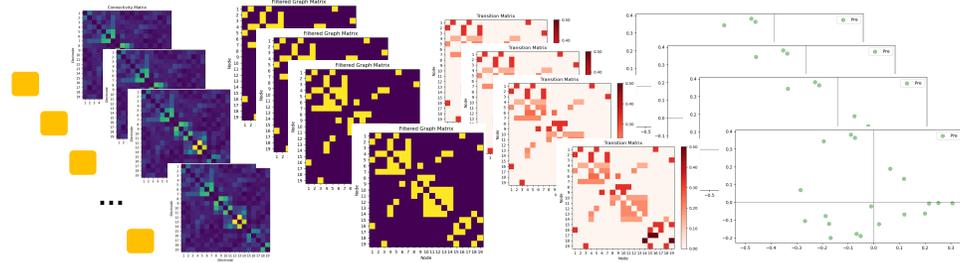
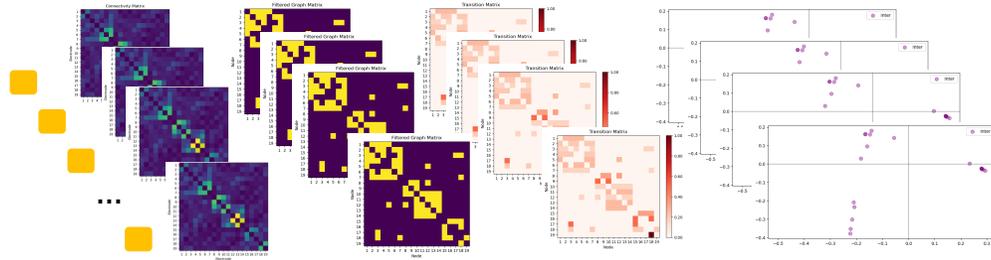
Data

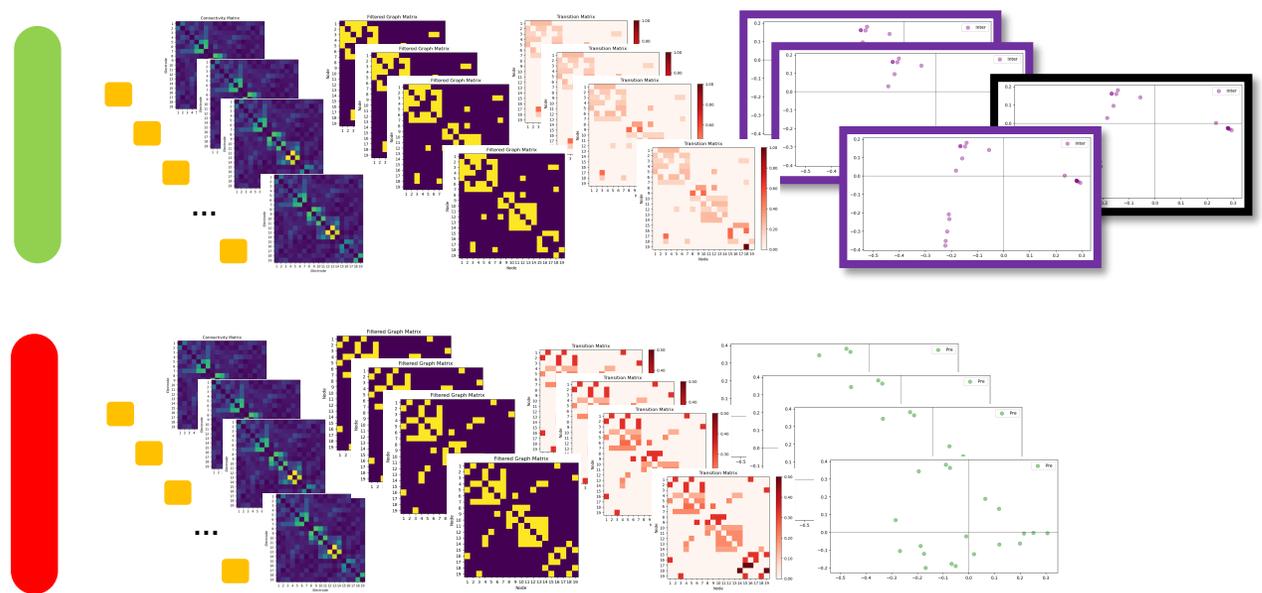
Diffusion Map

Procrustes

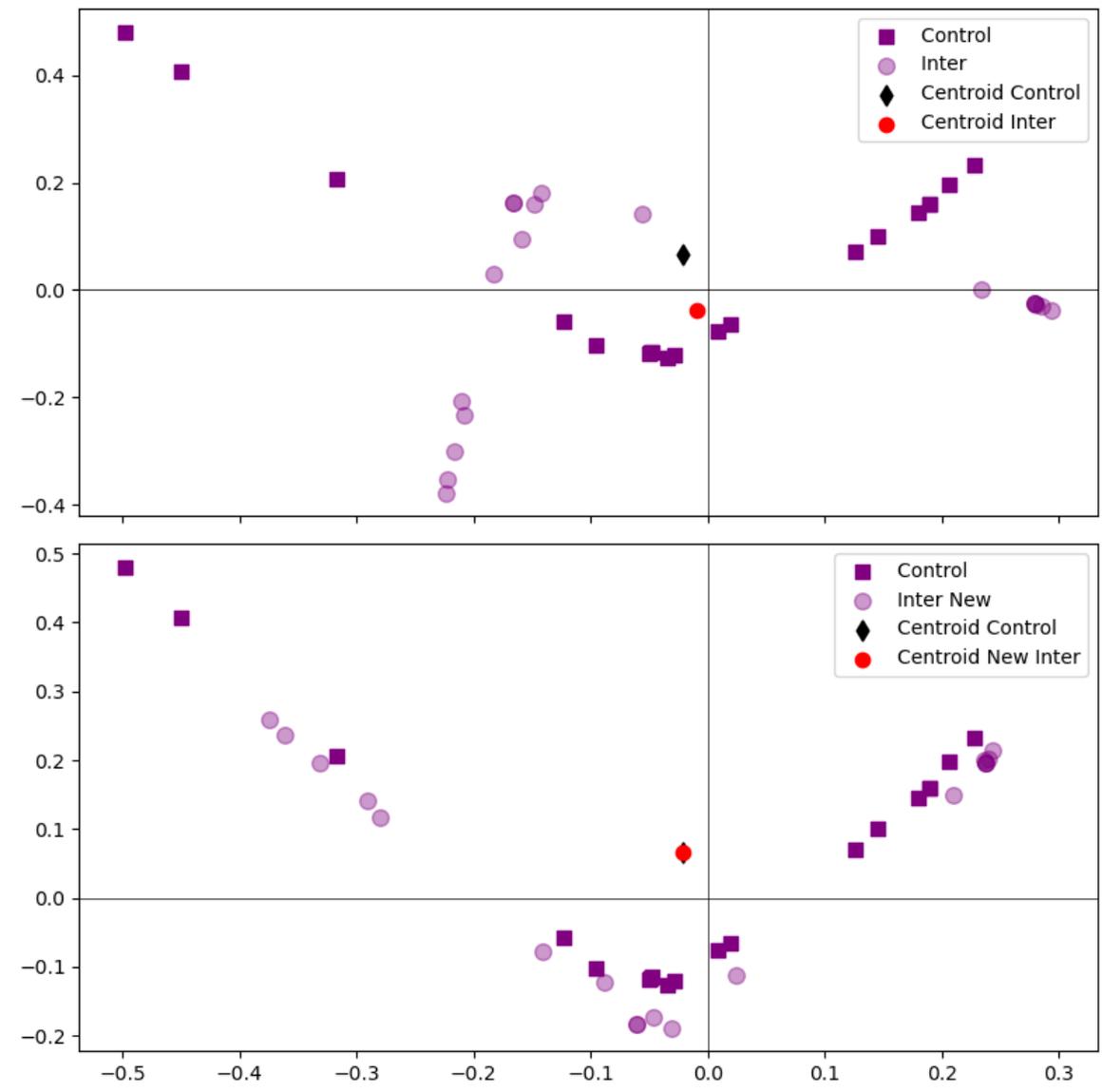
Distances

Nodes

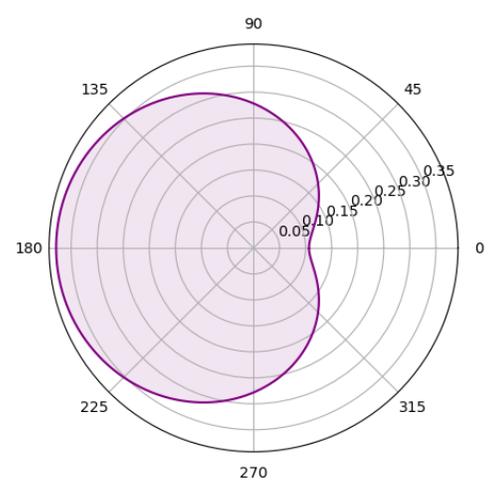


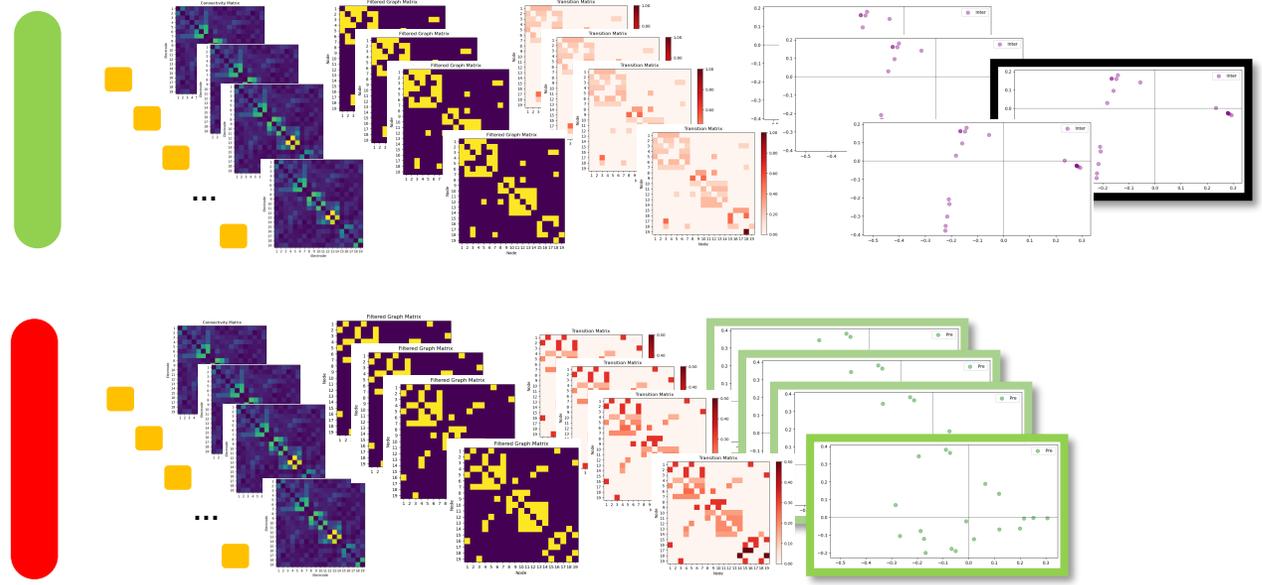


2D Spatial Distribution of the Embedding in Euclidean Space

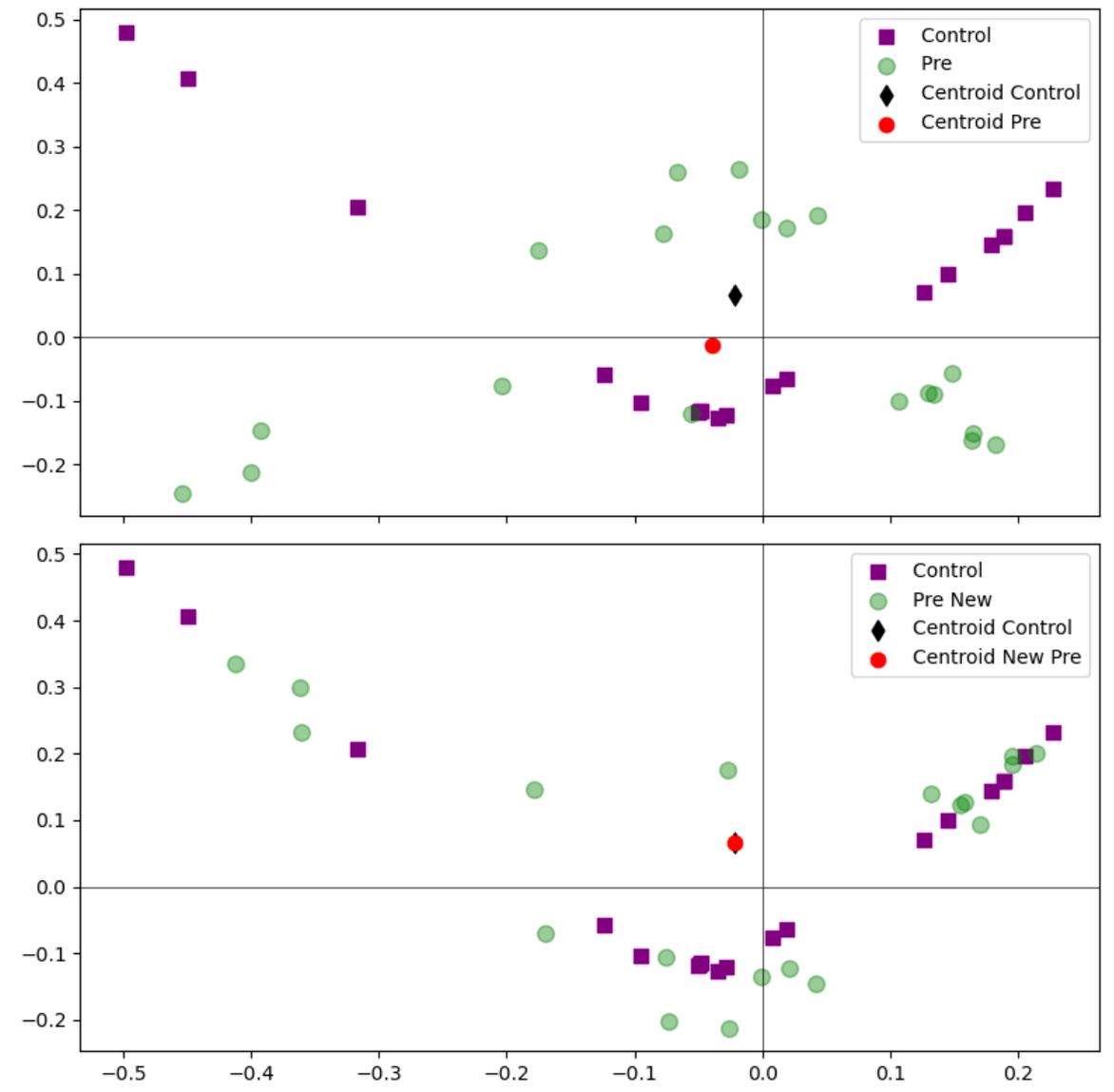


Final alignment's RMS values on rotation

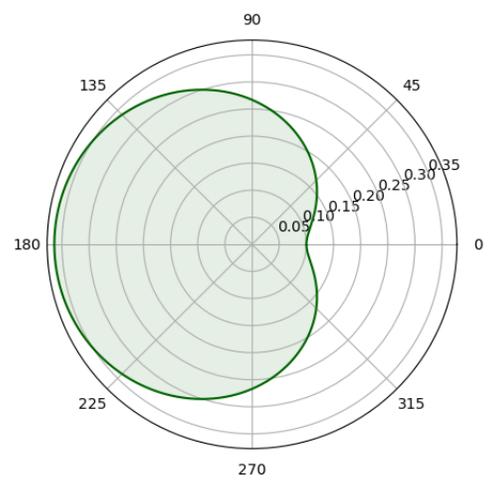




2D Spatial Distribution of the Embedding in Euclidean Space



Final alignment's RMS values on rotation





Data

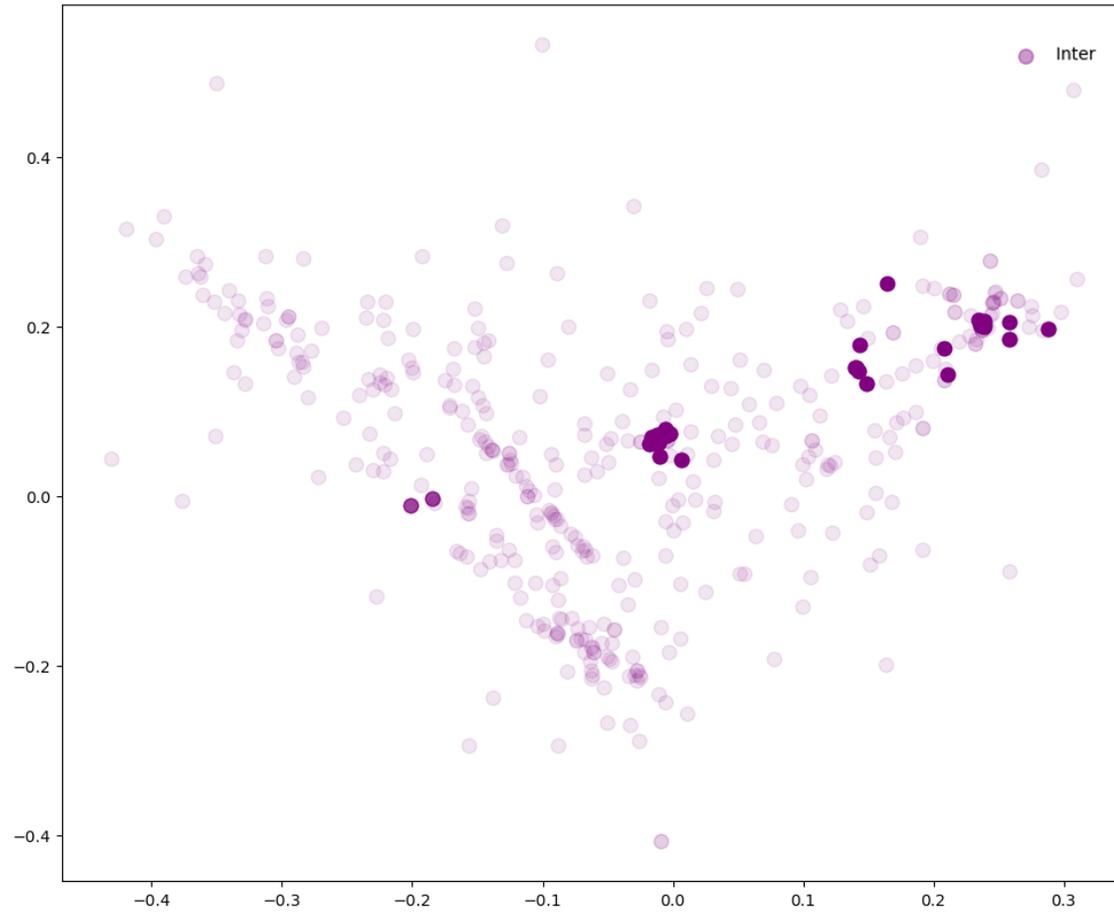
Diffusion Map

Procrustes

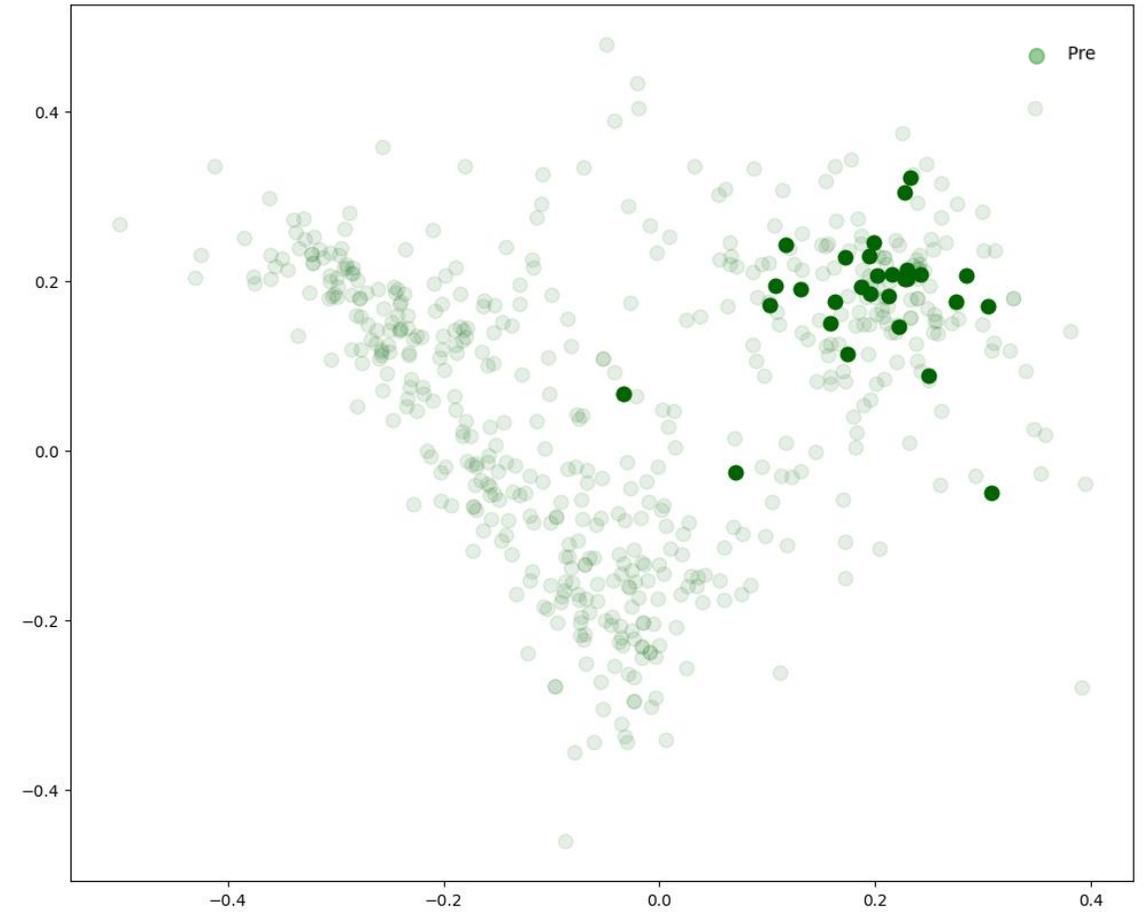
Distances

Nodes

Spatial Distribution of the Complete data Embedding

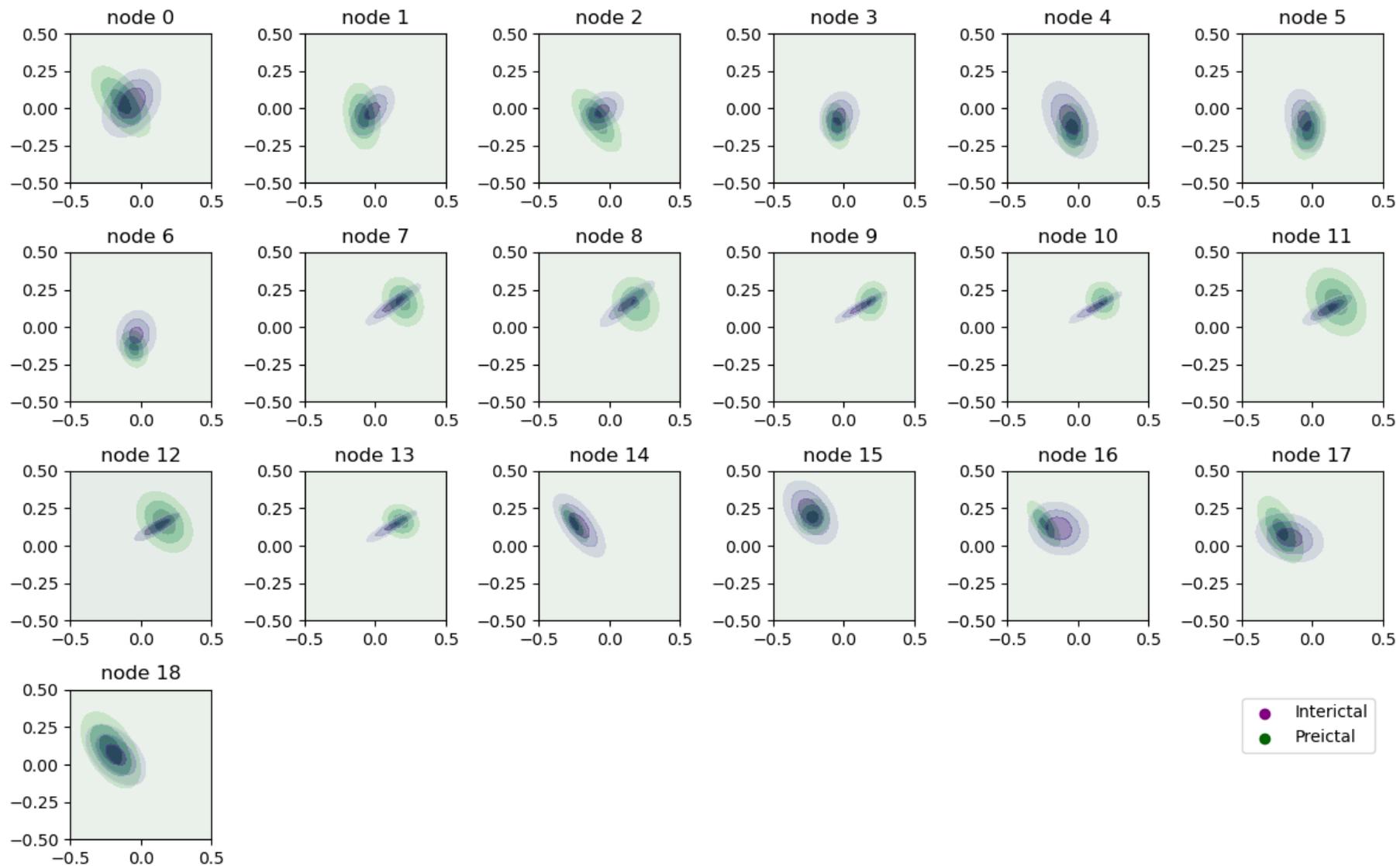


Spatial Distribution of the Complete data Embedding





Distribution of the Spatial Position of the Data Node by Node

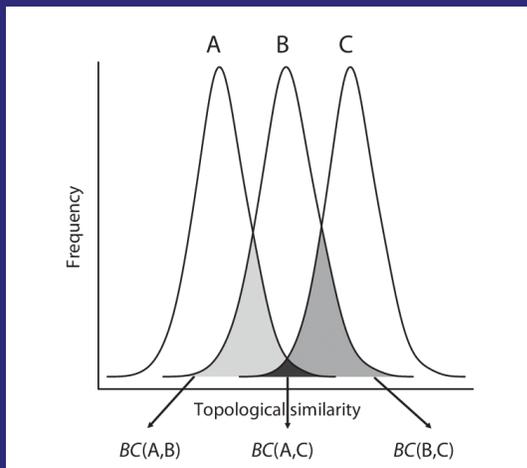




Metrics to measure the distance between two probability distributions

### Bhattacharyya

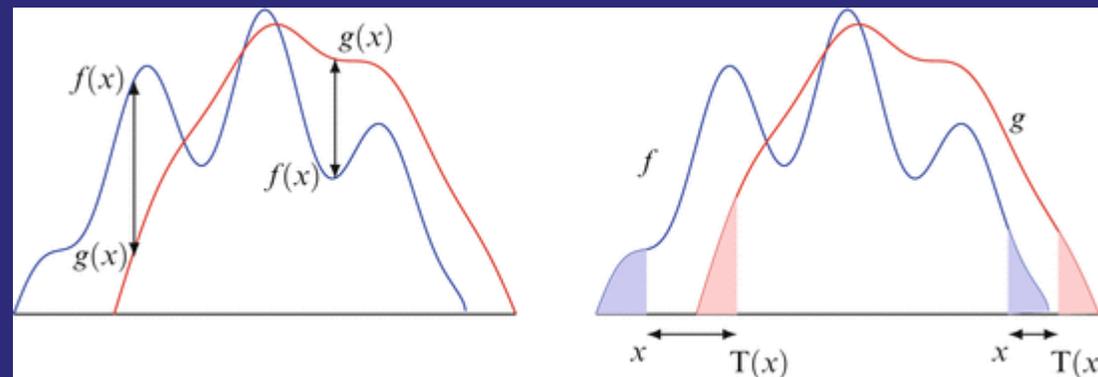
$$D_B(p_1, p_2) = \frac{1}{8} (\mu_1 - \mu_2)^T \Sigma^{-1} (\mu_1 - \mu_2) + \frac{1}{2} \ln \left( \frac{\det \Sigma}{\sqrt{\det \Sigma_1 \det \Sigma_2}} \right)$$

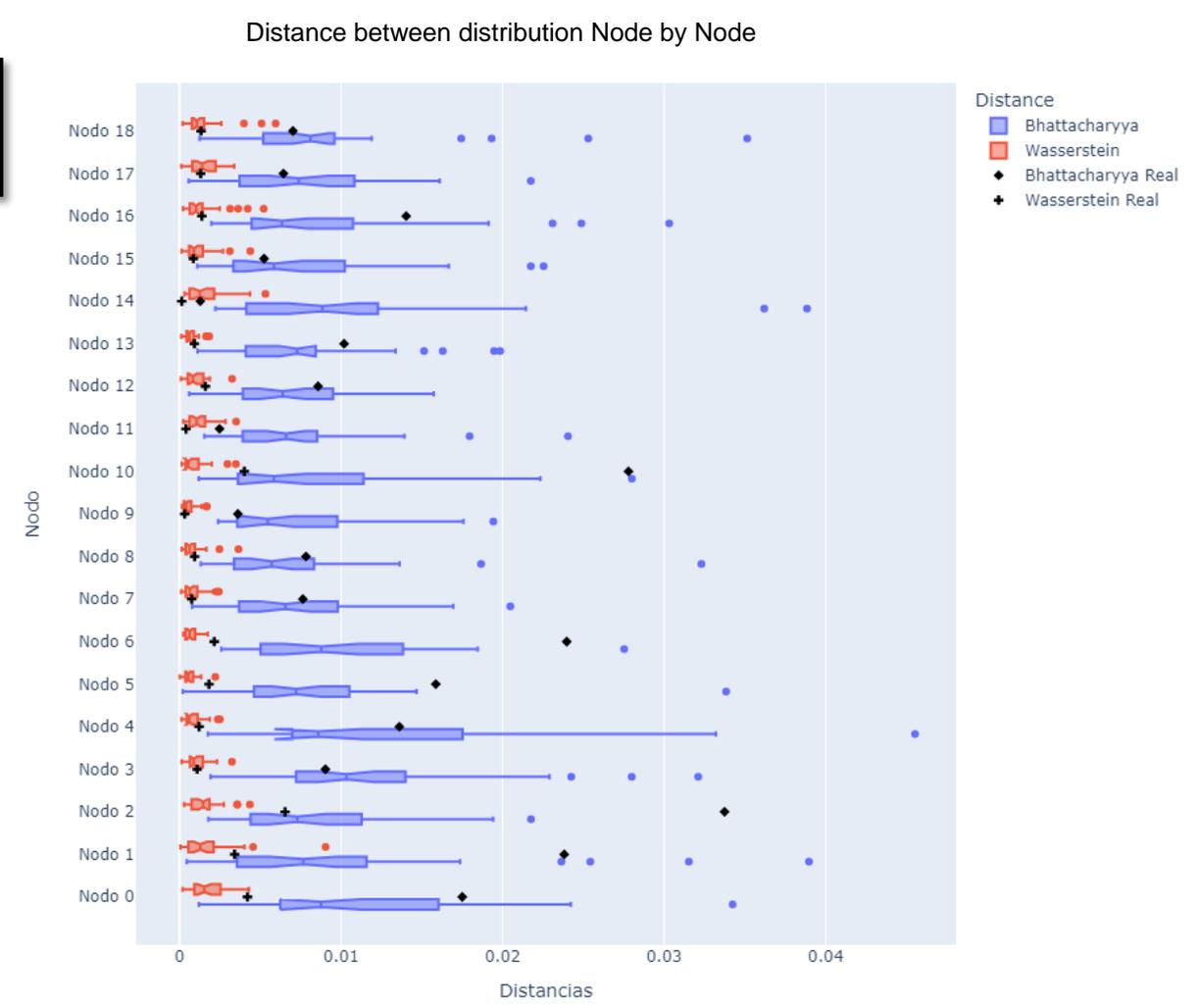
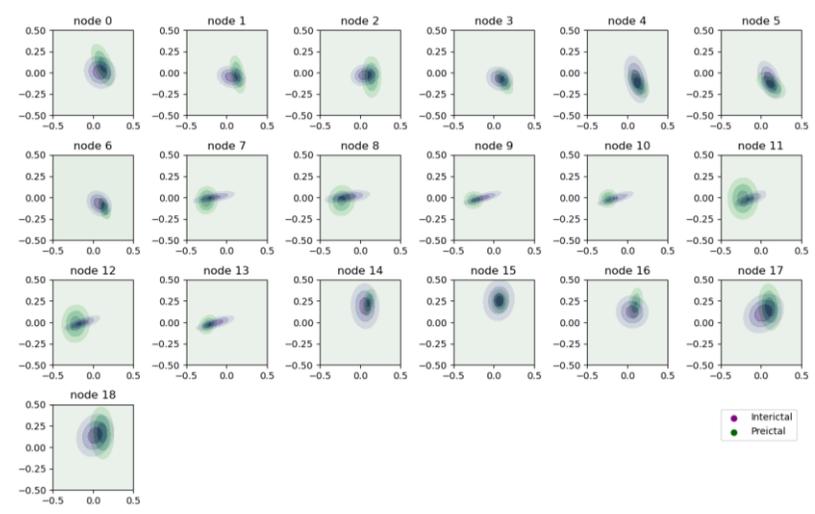
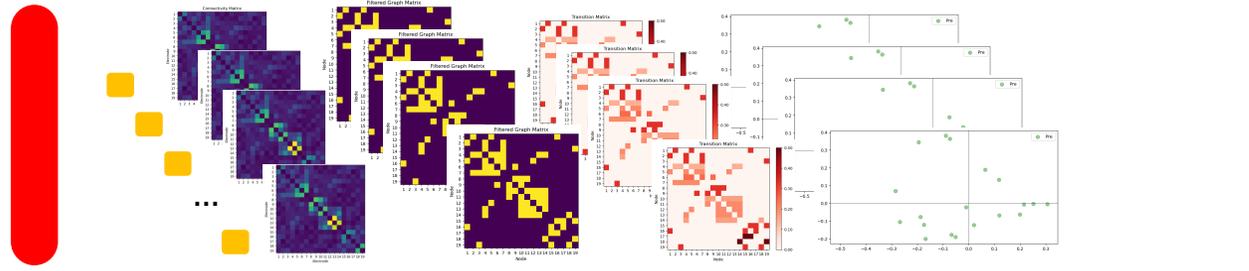
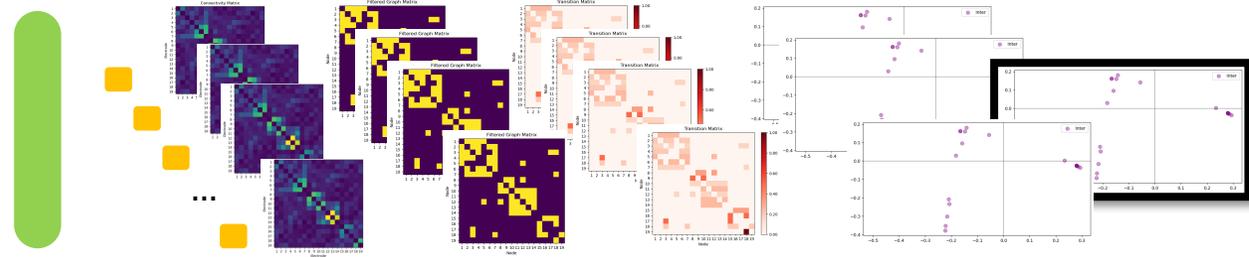


## Distances

### Wasserstein

$$D_W = \|m_1 - m_2\|^2 + \text{trace} \left( C_1 + C_2 - 2 \left( C_2^{\frac{1}{2}} C_1 C_2^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)$$





The random test (shuffling) is to defined the base line

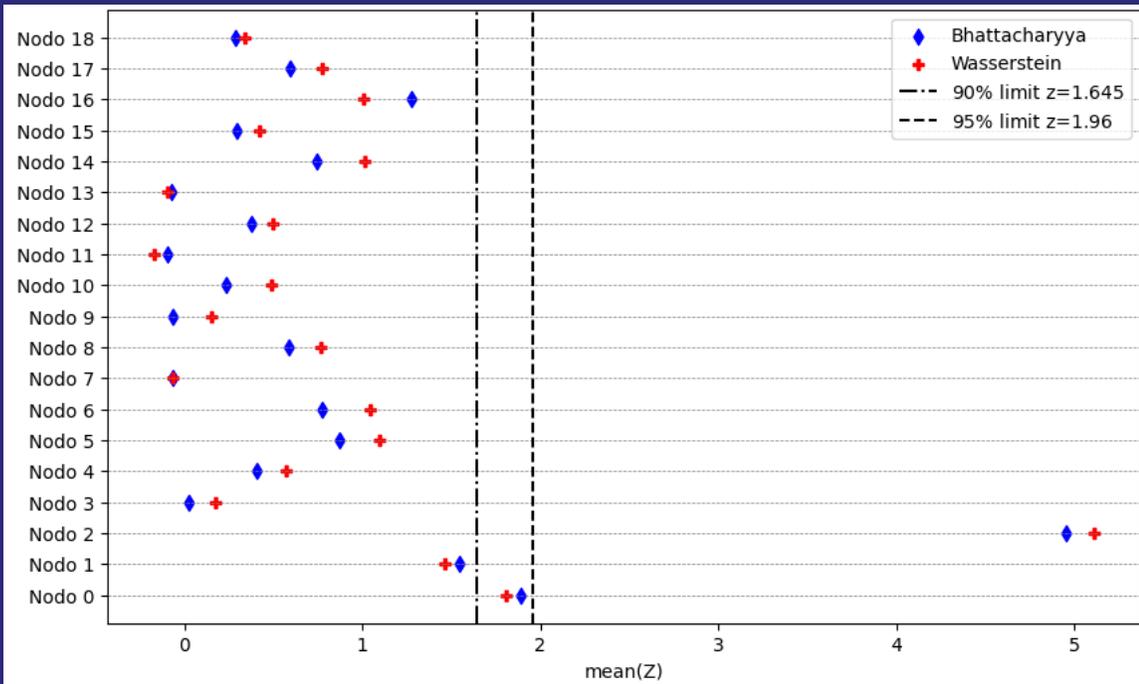


# Nodes

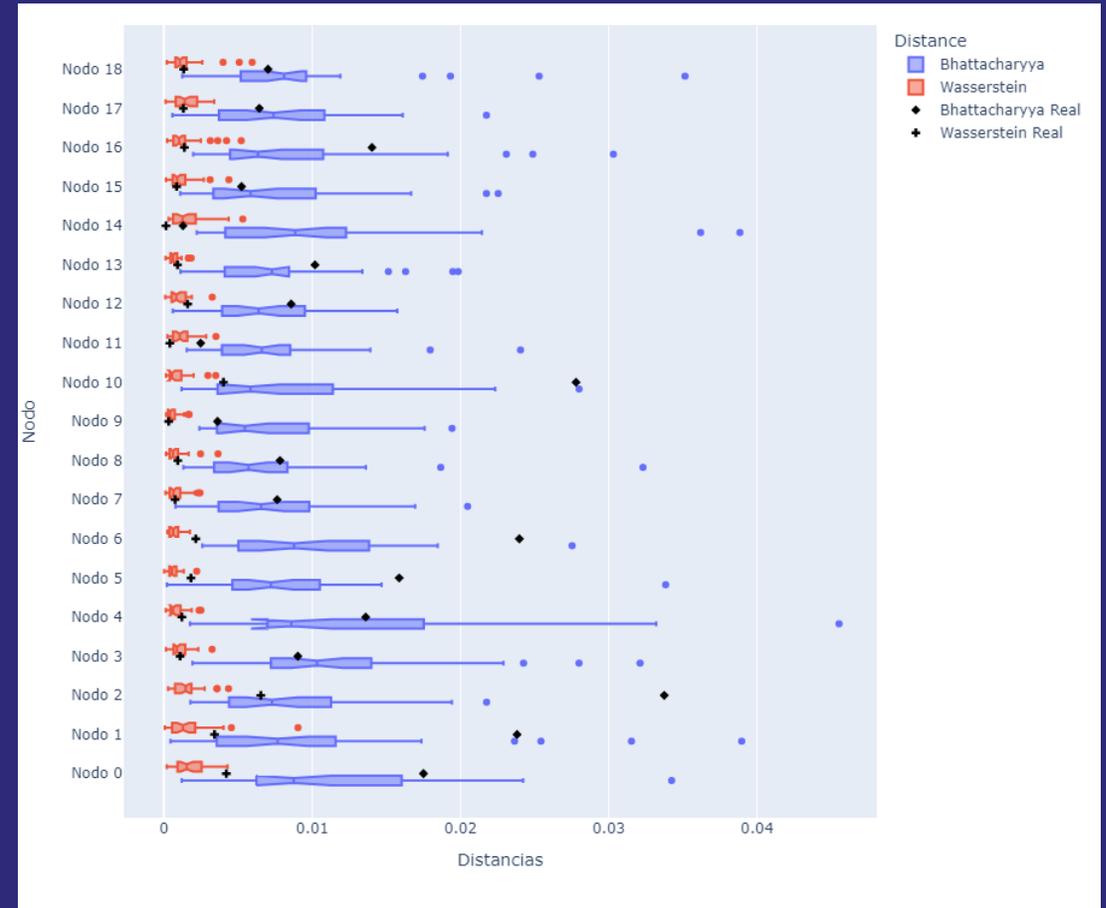
Measurements normalized by z scores

$$Z = \frac{x - \mu}{\sigma}$$

Z scores for each node



Distance between distribution Node by Node





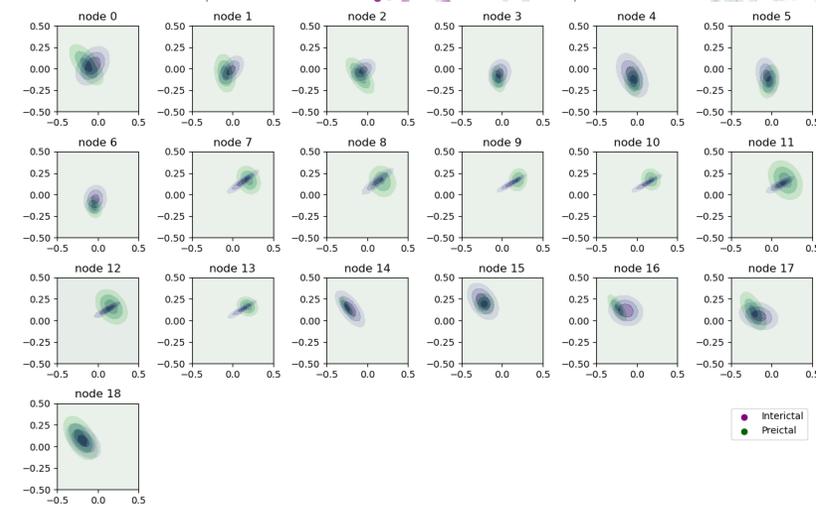
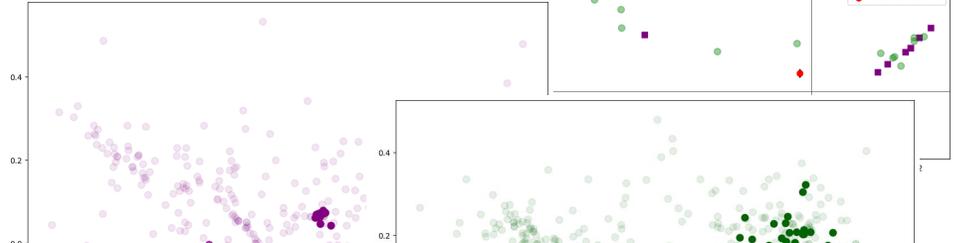
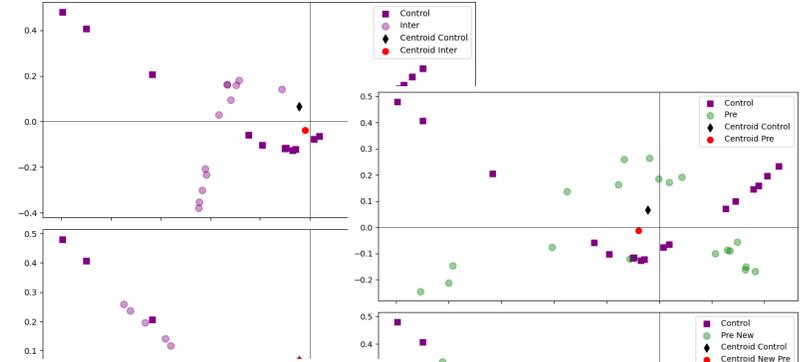
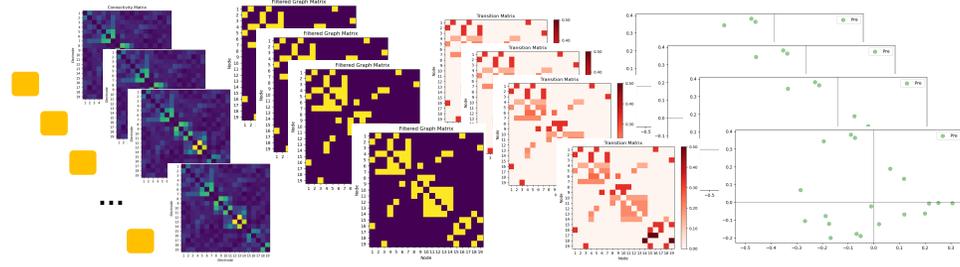
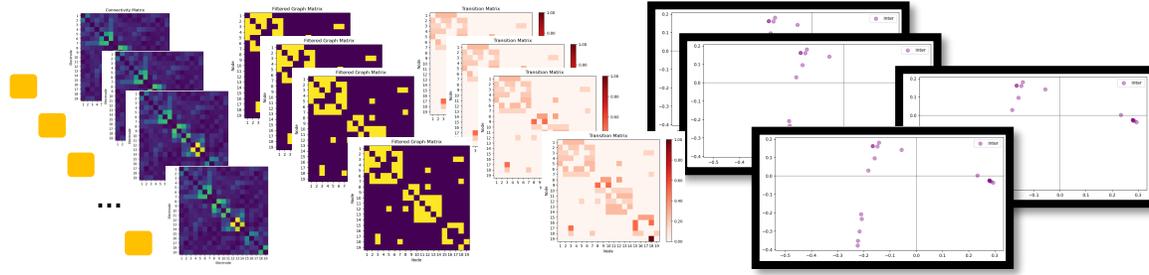
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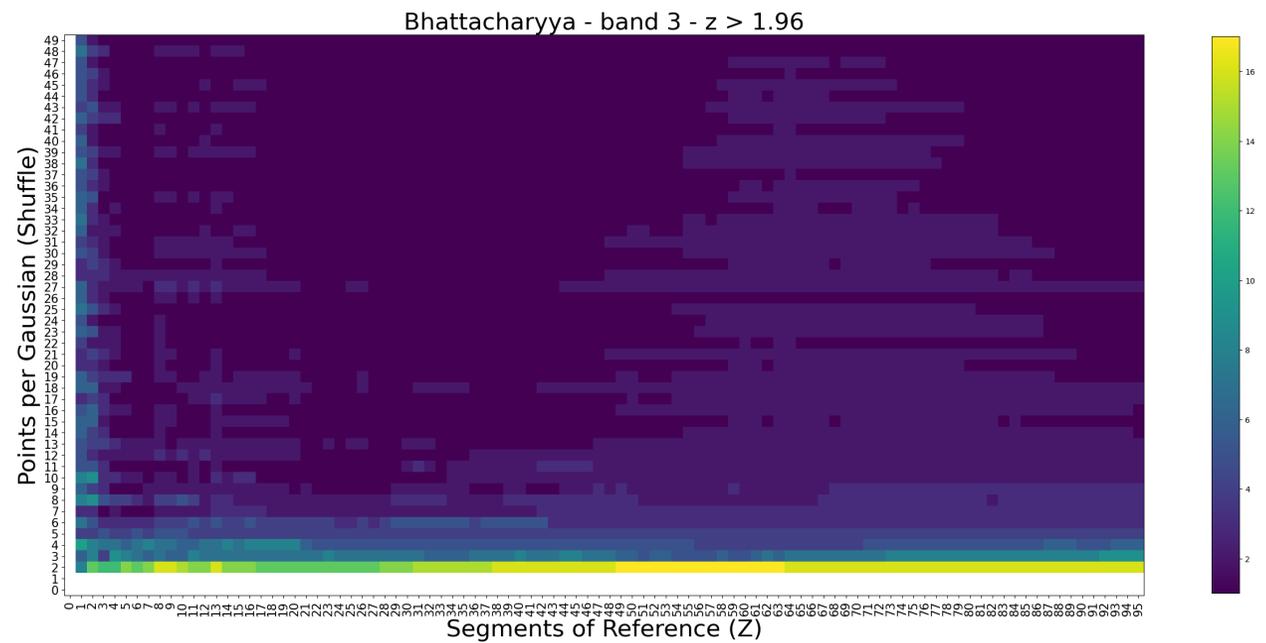
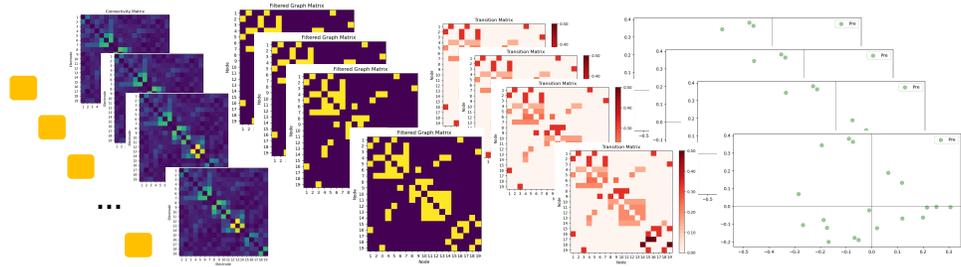
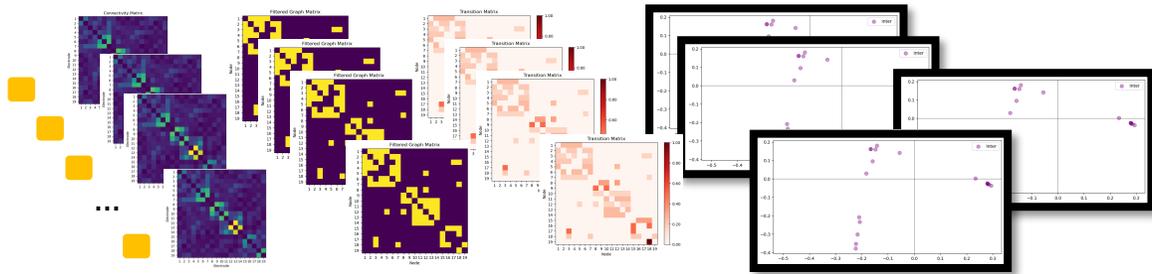
# Diffusion Map

# Procrustes

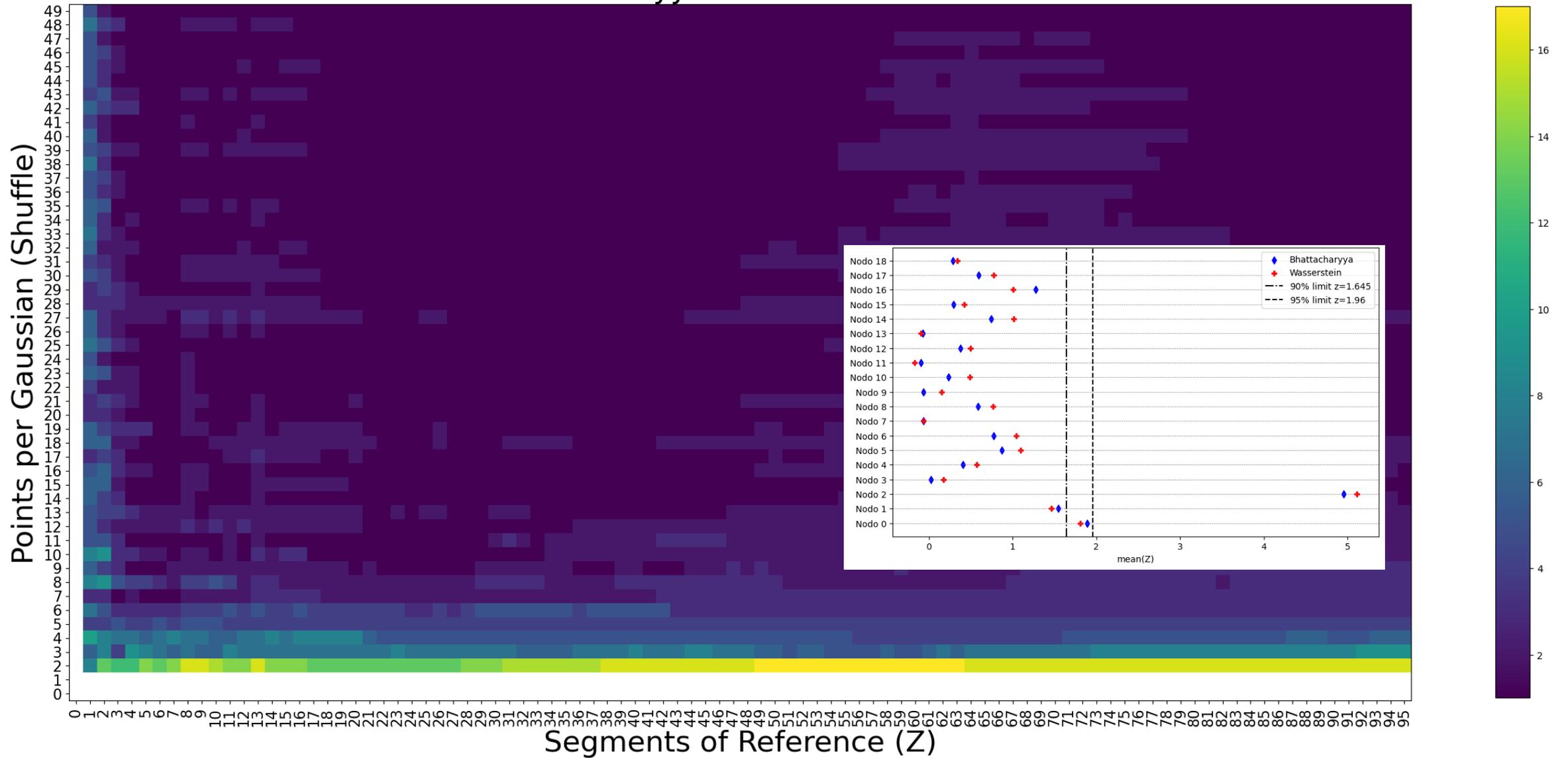
# Distances

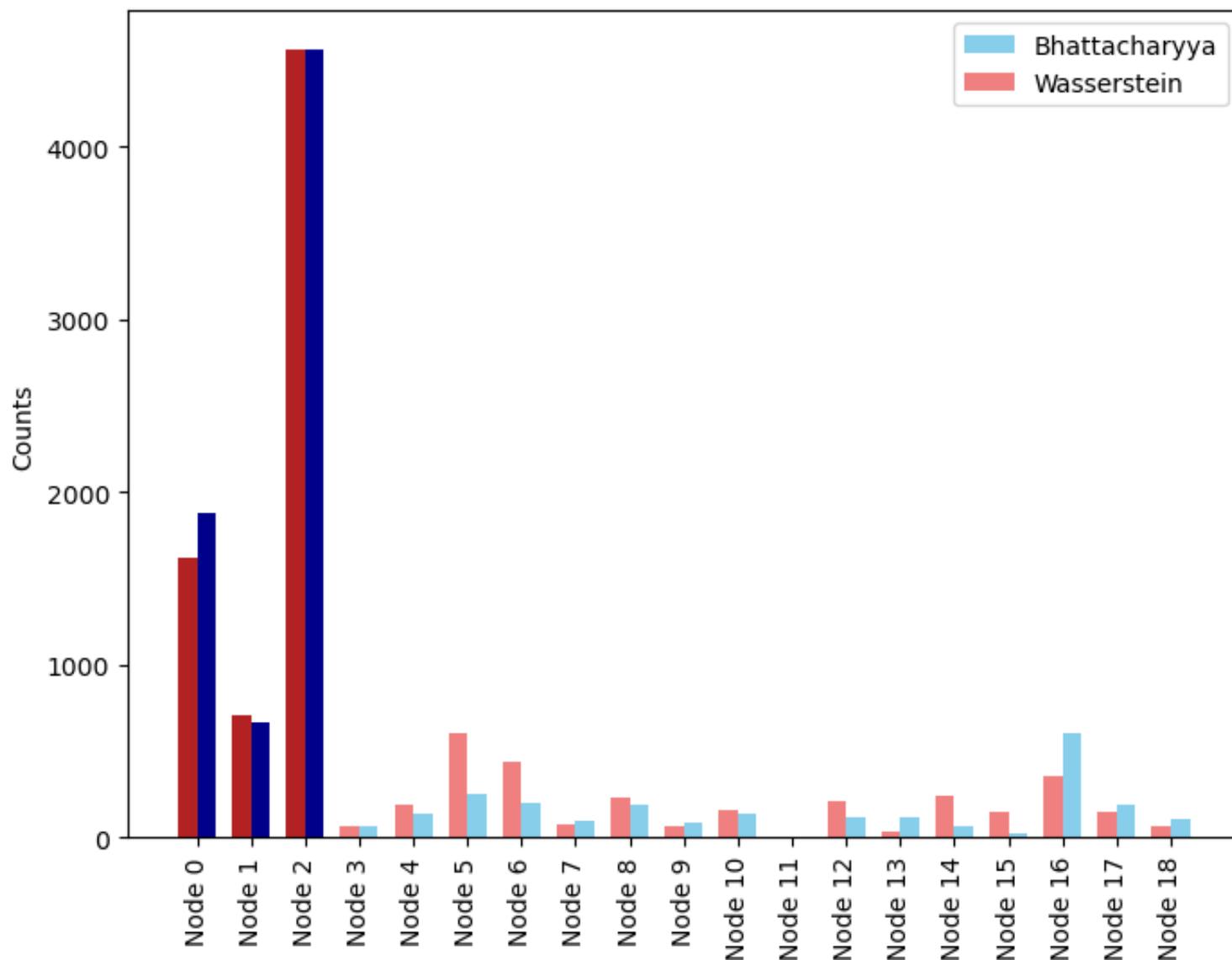
# Nodes





### Bhattacharyya - band 3 - $z > 1.96$





## Conclusion

In a first approach, embedding iEEG data in the Euclidean space of diffusion maps allows us to identify local connectivity patterns in the network nodes, highlighting those with potential to differentiate seizure states in epilepsy patients.

## Impact

This is information that is of interest to clinical staff. We can go back to the brain and point out the region of interest where something is happening. We can also correlate if our node corresponds to the center of the seizure event.



<http://laconga.redclara.net>



[contacto@laconga.redclara.net](mailto:contacto@laconga.redclara.net)



lacongaphysics



Latin American alliance for  
Capacity buildiNG in Advanced physics

**LA-CoNGA physics**



Cofinanciado por el  
programa Erasmus+  
de la Unión Europea

El apoyo de la Comisión Europea para la producción de esta publicación no constituye una aprobación del contenido, el cual refleja únicamente las opiniones de los autores, y la Comisión no se hace responsable del uso que pueda hacerse de la información contenida en la misma.



# EPILEPSIA:

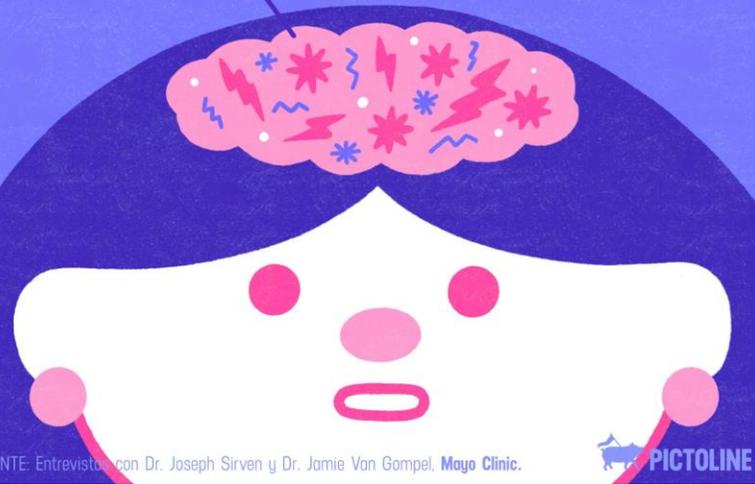
TODO LO QUE NECESITAS SABER SOBRE ESTA CONDICIÓN

## ¿QUÉ ES?

UNA ALTERACIÓN DE LA ACTIVIDAD ELÉCTRICA DEL CEREBRO QUE PUEDE SER GENERALIZADA O SOLO EN UNA REGIÓN.

SUS CAUSAS SON VARIADAS: FACTORES GENÉTICOS, INFECCIONES, TUMORES, HEMORRAGIAS O LESIONES.

Y ES TAN COMÚN QUE SI EN UNA HABITACIÓN HAY 20 PERSONAS, ES MUY PROBABLE QUE AL MENOS 1 LA PADEZCA.



FUENTE: Entrevistas con Dr. Joseph Sirven y Dr. Jamie Van Gampel. Mayo Clinic.



## SÍNTOMAS

SUELE MANIFESTARSE EN FORMA DE CONVULSIONES



PERO TAMBIÉN COMO ESPASMOS EN UNA PARTE DEL CUERPO



CRISIS DE AUSENCIA



(LUCE COMO SI ESTUVIERA MIRANDO A LA NADA POR UNOS SEGUNDOS)

MOVIMIENTOS REPETITIVOS (TICS)



SENSACIONES INUSUALES (COMO PERCIBIR LUCES, OLORES, ETC.)



PÉRDIDA DE CONSCIENCIA



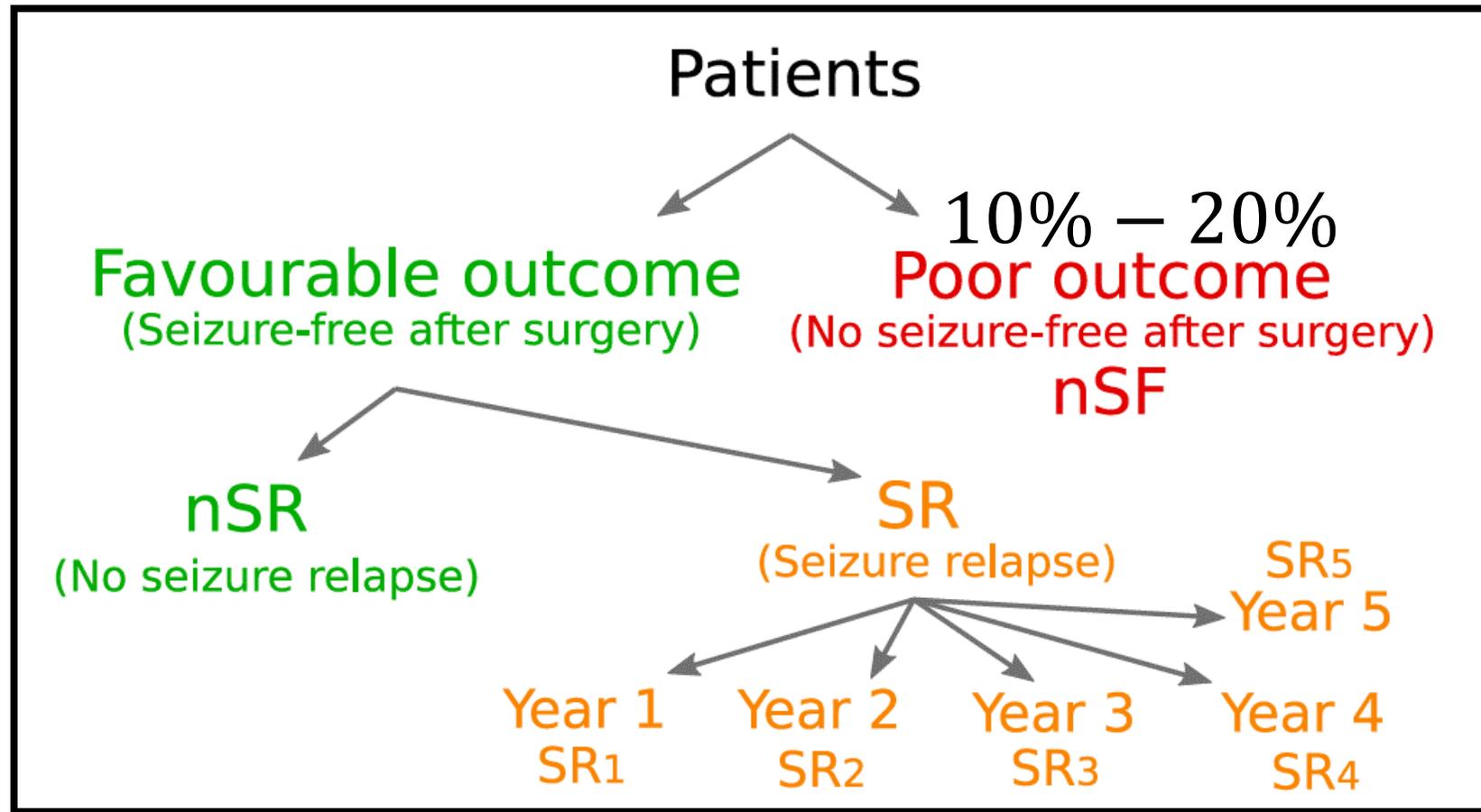
## ¿Y QUÉ HACER SI SOSPECHAS QUE LA TIENES?

TÍPICAMENTE LA DIAGNOSTICA UN ESPECIALISTA EN NEUROLOGÍA.

VE A LA CONSULTA CON ALGUIEN QUE TE HAYA VISTO CON LOS SÍNTOMAS, YA QUE ES PROBABLE QUE NO RECUERDES LO QUE PASÓ.

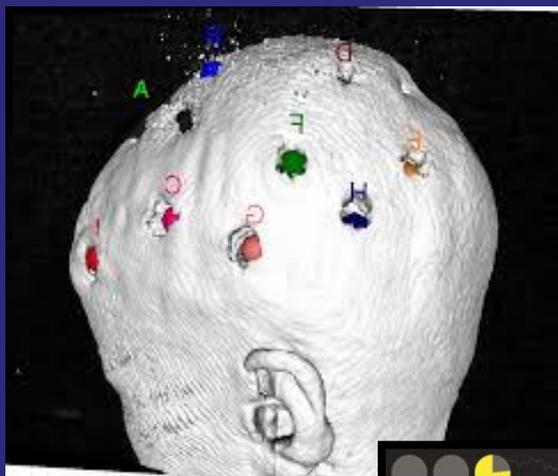
PARA LA MAYORÍA DE LAS PERSONAS EXISTE ALGUNA SOLUCIÓN: DESDE MEDICAMENTOS, DIETAS O USO DE MARCAPASOS HASTA CIRUGÍA



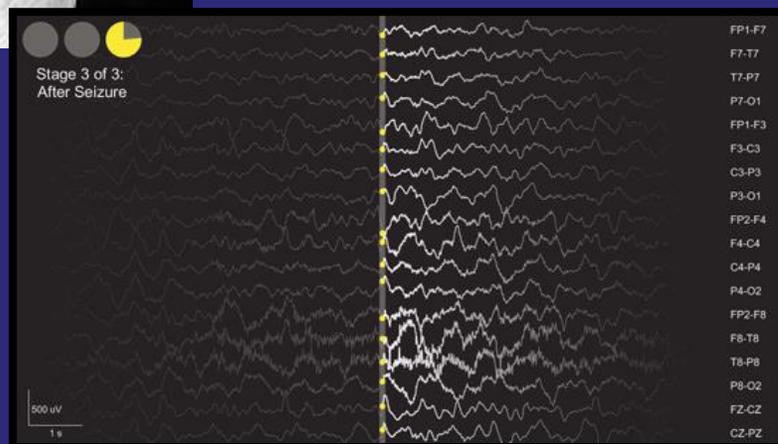




## Data

intracranial electroencephalography  
(iEEG)

method to record an electrogram of the spontaneous electrical activity of the brain. EEG is the gold standard diagnostic procedure to confirm epilepsy. The sensitivity of a routine EEG to detect interictal epileptiform discharges at epilepsy centers has been reported to be in the range of 29–55%



Delta &lt; 4 Hz

Theta 4 – 7 Hz

Alpha 8 – 12 Hz

Beta 13 – 30 Hz

Low Gamma

High Gamma

&gt; 30 Hz



Diffusion Map

Procrustes

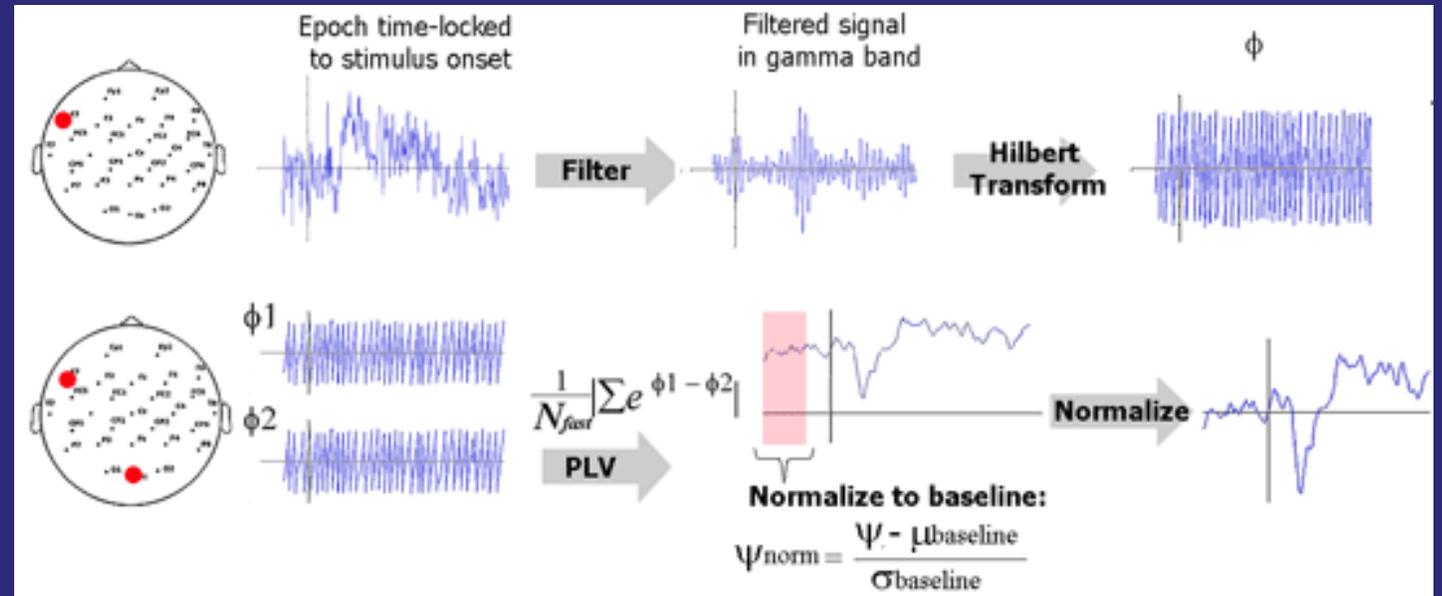
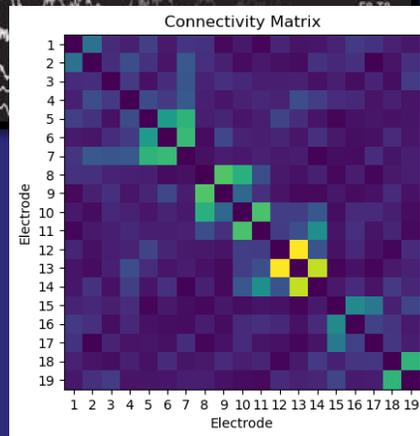
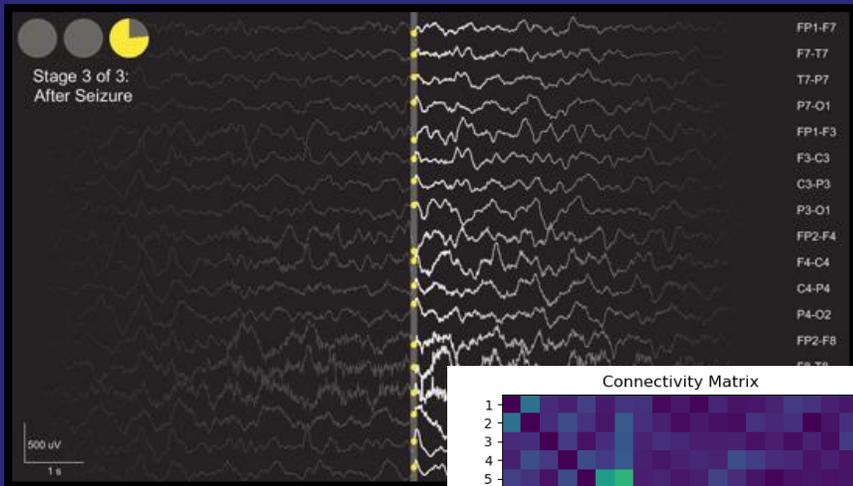
Distances

Nodes

# Data

intracranial electroencephalography  
(iEEG)

Use of the Phase Locking Value Method  
(PLV) to construct the connectivity matrixes  
with the iEEG signals





Diffusion Map

Procrustes

Distances

Nodes

# Data



Band Beta

Interictal 8 days

Preictal 3 days



Band Alpha

Interictal 10 days

Preictal 2 days



Band High Gamma

Interictal 2 days

Preictal 2 days



Day

1

2

3

4

...

$m$

Time of data collection by iEEG



$0_{min}$

$10_{min}$



Interictal

Preictal

Band Low Gamma  
Interictal 9 days

Preictal 3 days



# Diffusion Map

Nonlinear dimensionality reduction technique of a data set in Euclidean space whose coordinates can be computed from the eigenvectors and eigenvalues of a diffusion operator on the data.

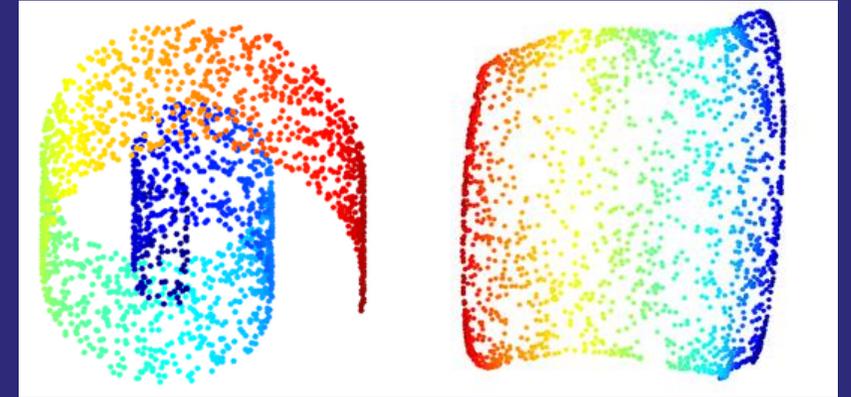
This reduction technique works by using the degree of the graph to find the probability of transition between all the data, creating a connection with the spectral theory of random walk.

$$p(x, y) = \frac{w(x, y)}{\sum_{z \in \Omega} w(x, z)}$$

$$\phi_j^T P = \lambda_j \phi_j^T \text{ and } P \psi_j = \lambda_j \psi_j$$

The eigenvalues and eigenvectors of the transition matrix ( $P$ ) provide the representation of the initial graph as a cloud of points in a lower dimensional space.

$$\Psi_t: x \rightarrow \begin{pmatrix} \lambda_1^t \psi_1(x) \\ \lambda_2^t \psi_2(x) \\ \vdots \\ \lambda_{q(t)}^t \psi_{q(t)}(x) \end{pmatrix}$$



Shan, S., & Daubechies, I. (2022). Diffusion maps: Using the semigroup property for parameter tuning



Data

Diffusion Map

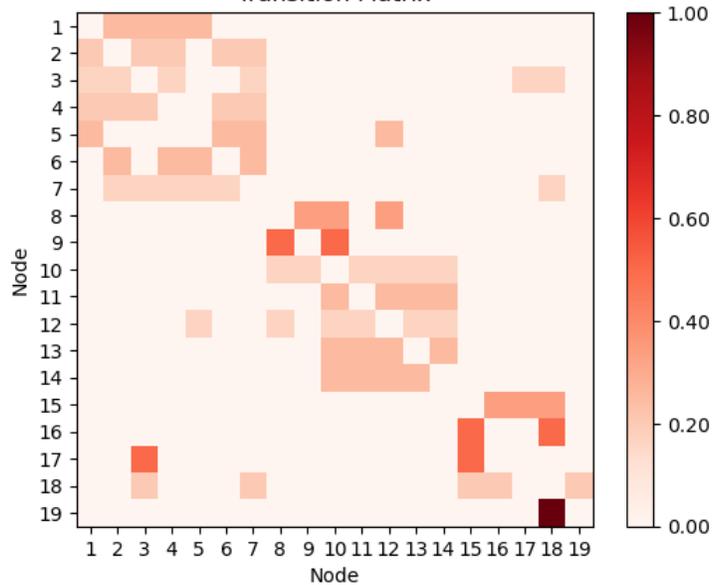
Procrustes

Distances

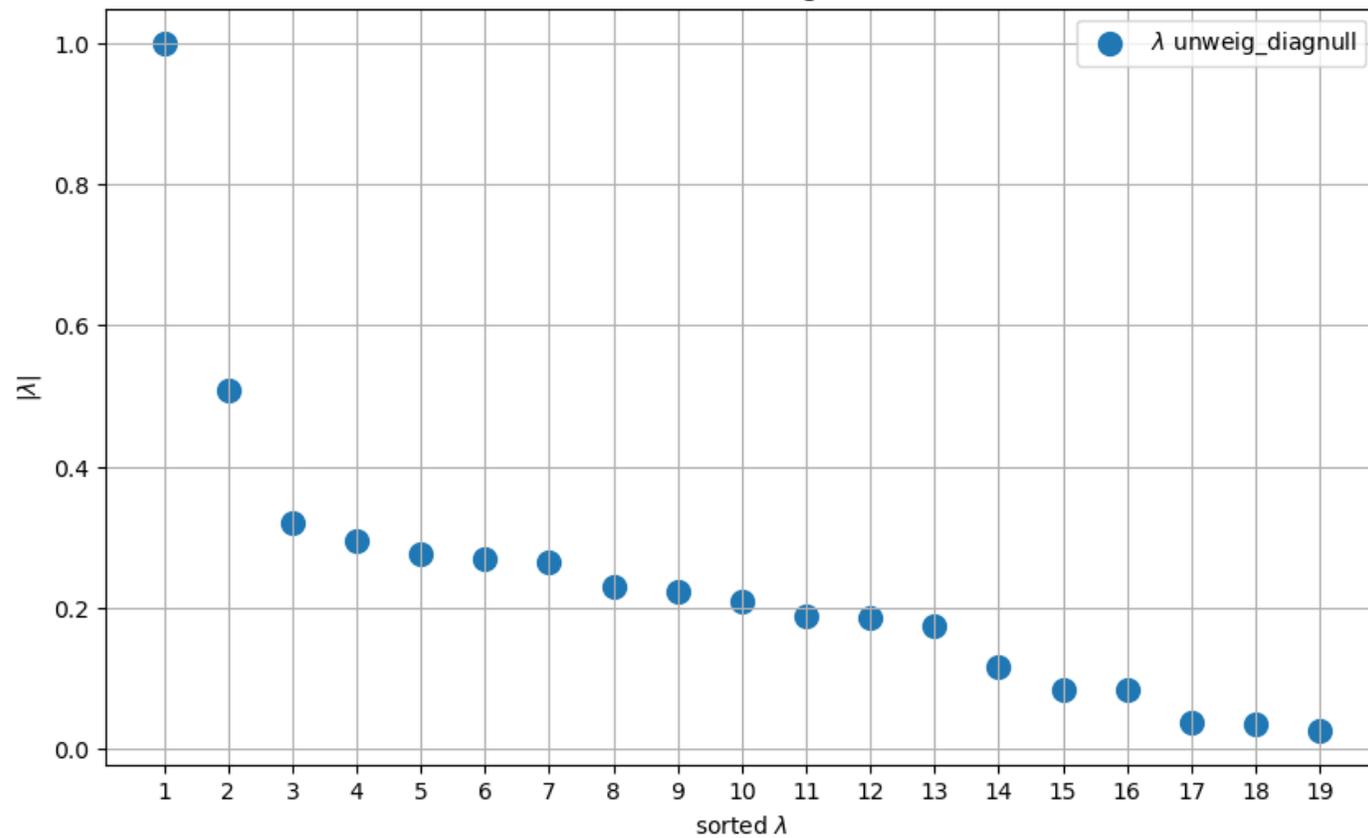
Nodes



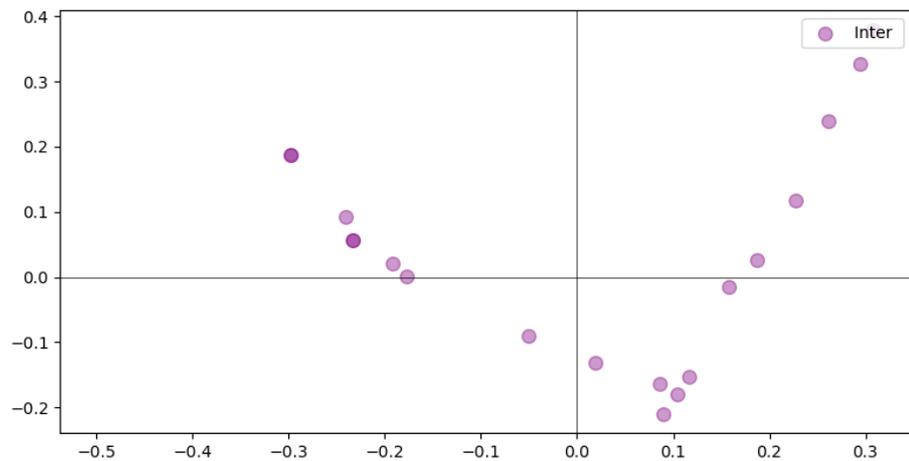
Transition Matrix



Transition Matrix Eigenvalues



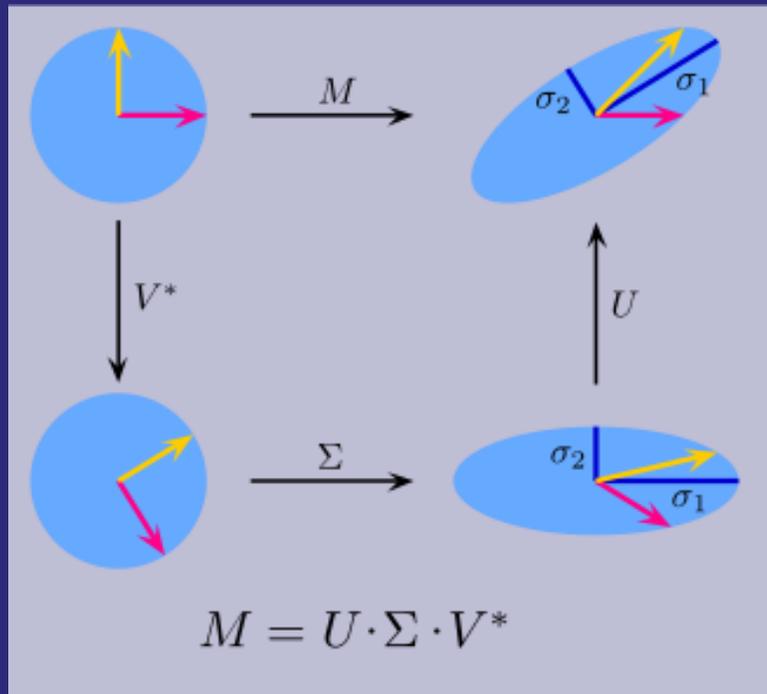
2D Spatial Distribution of the Embedding in Euclidean Space





# Procrustes

Generalized Procrustes Analysis (GPA) is a multivariate exploratory technique that involves transformations (i.e., translation, rotation, reflection and isotropic rescaling) of individual data matrices to provide optimal comparability.



First, it is necessary to compute the baricenter of the two dataset and translate the to the origin of the space.

This problem is equivalent to finding the nearest orthogonal matrix to a given matrix  $M = BA^T$  using the singular value decomposition

$$M = U \Sigma V^T$$

Where the transformation matrix is

$$R = UV^T$$

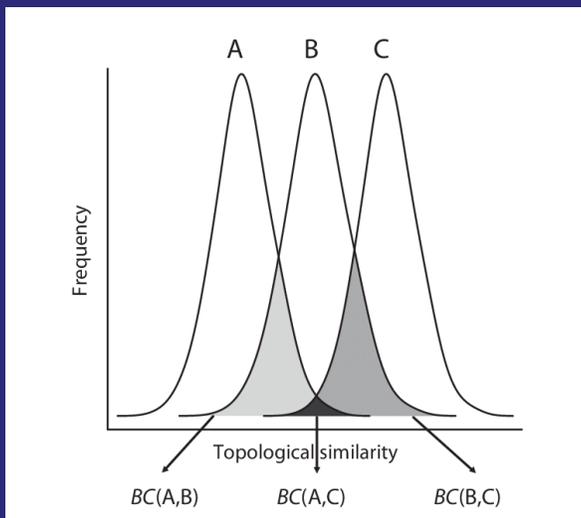


# Distances

## Bhattacharyya

$$D_B(p_1, p_2) = \frac{1}{8} (\mu_1 - \mu_2)^T \Sigma^{-1} (\mu_1 - \mu_2) + \frac{1}{2} \ln \left( \frac{\det \Sigma}{\sqrt{\det \Sigma_1 \det \Sigma_2}} \right)$$

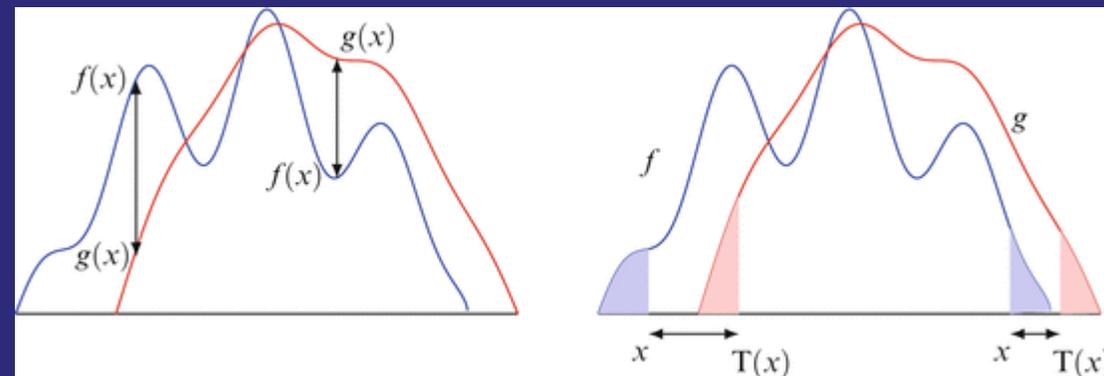
$$\Sigma = \frac{\Sigma_1 + \Sigma_2}{2}$$



It is based on the measure of overlapping areas under the probability curves of the two distributions.

## Wasserstein

$$D_W = \|m_1 - m_2\|^2 + \text{trace} \left( C_1 + C_2 - 2 \left( C_2^{\frac{1}{2}} C_1 C_2^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)$$



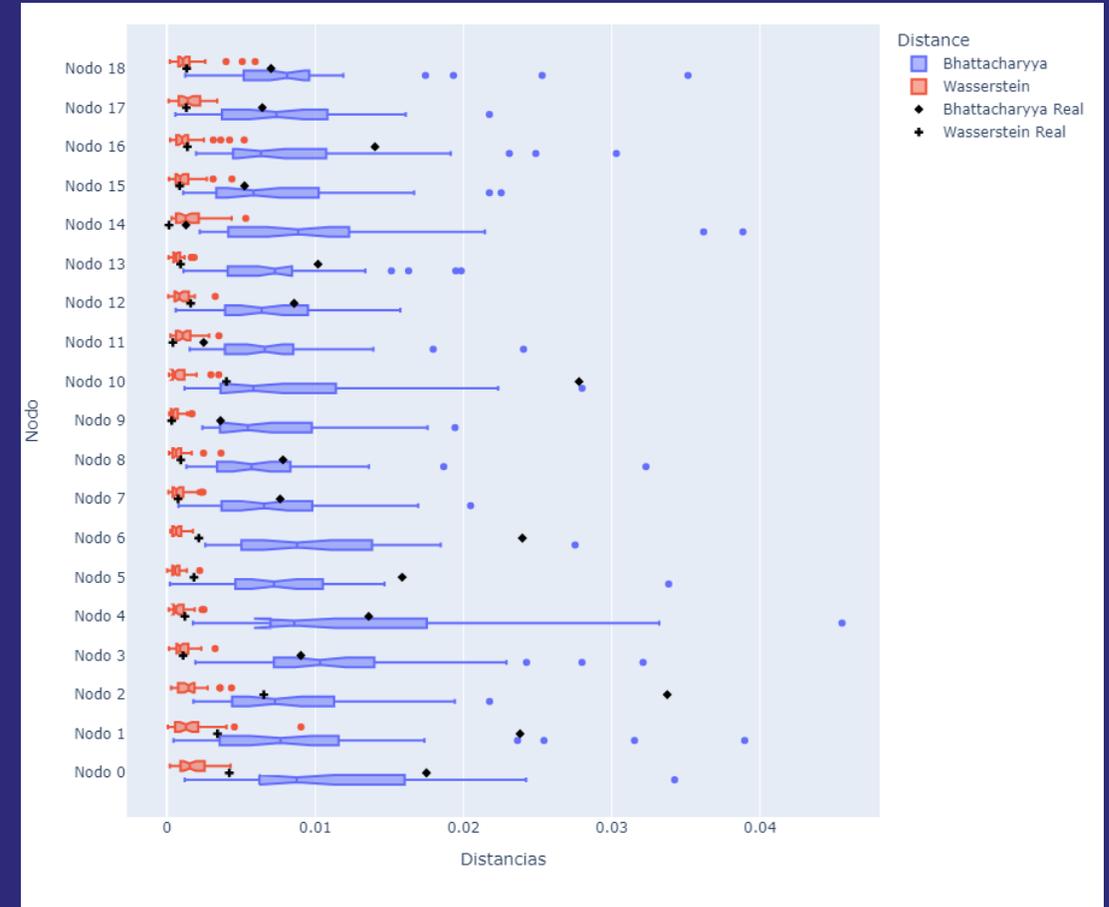
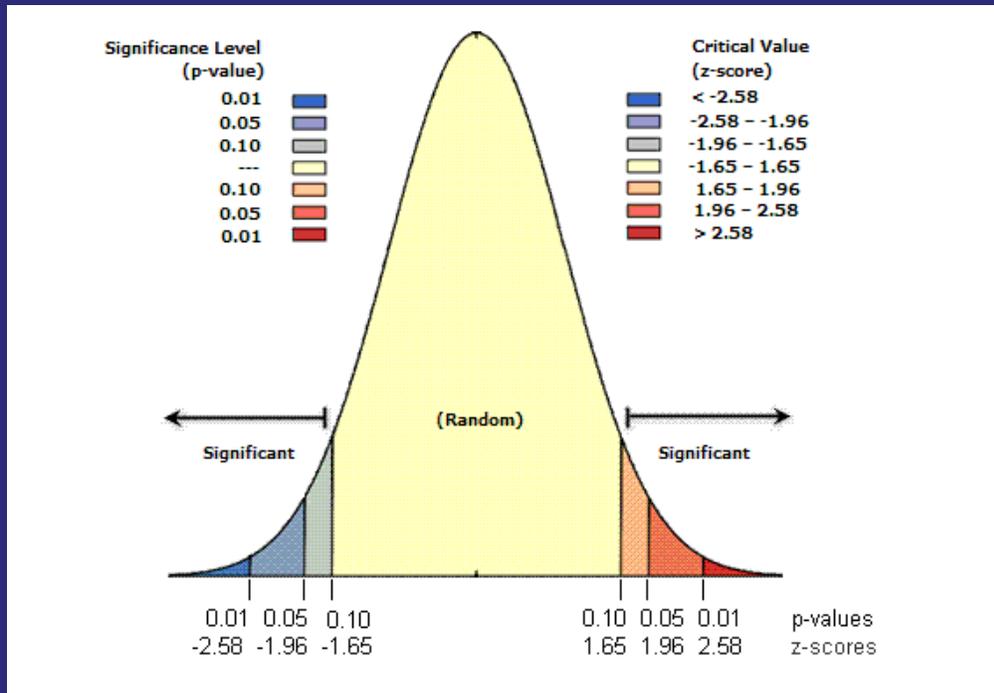
It measures the minimum "cost" necessary to transform one probability distribution into another.



# Nodes

Measurements normalized by z scores

$$Z = \frac{x - \mu}{\sigma}$$





Data

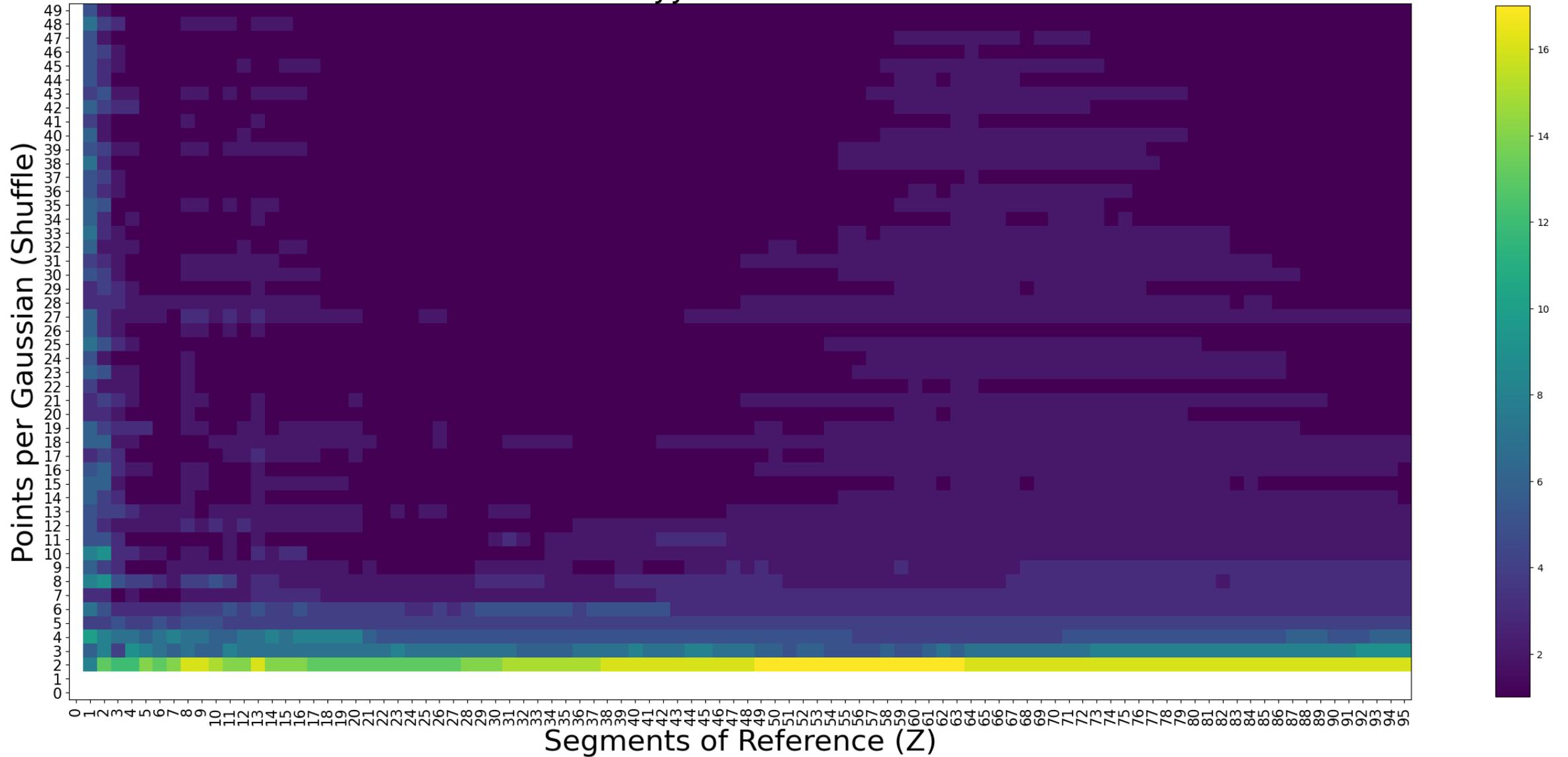
Diffusion Map

Procrustes

Distances

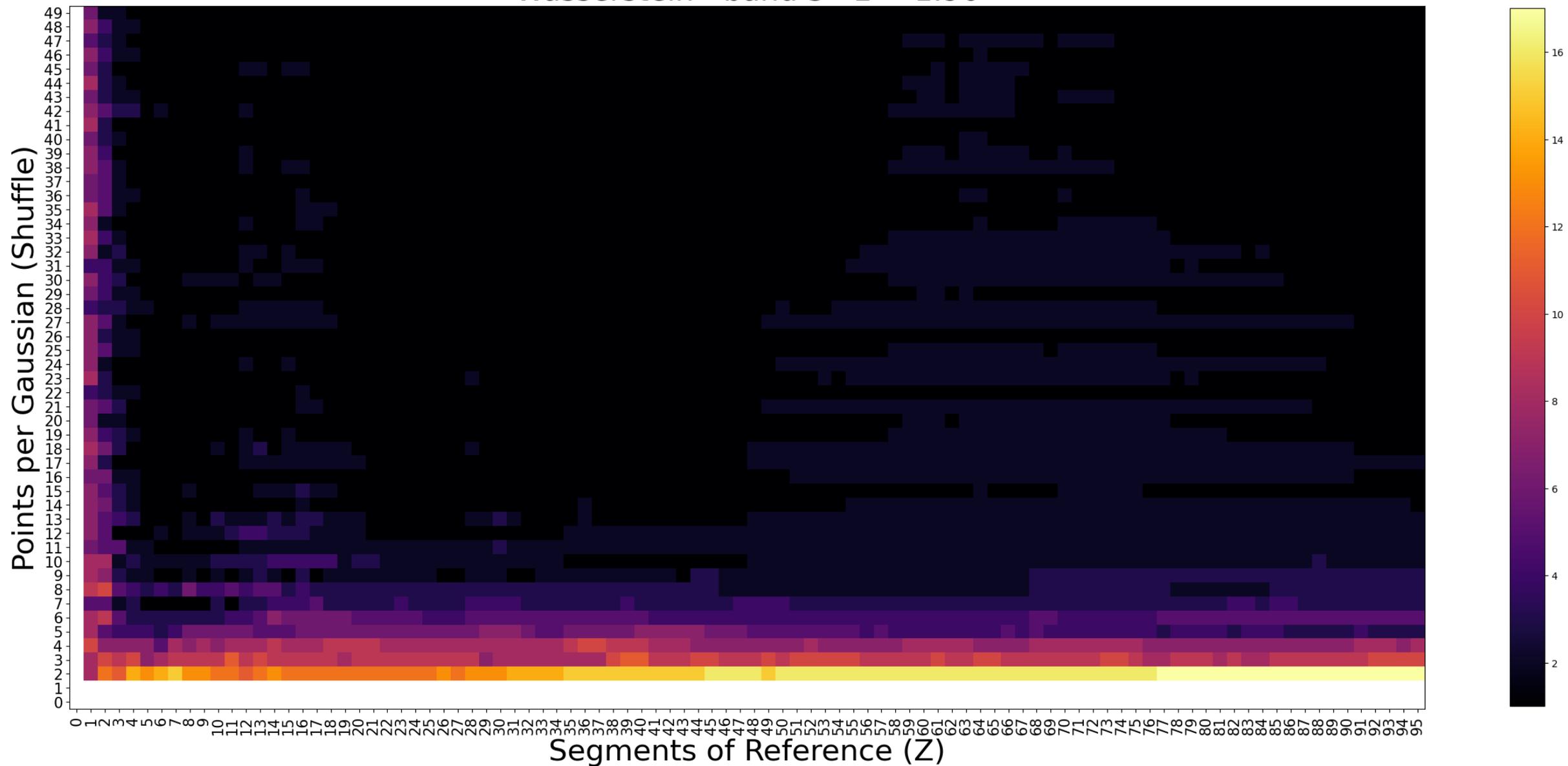
Nodes

Bhattacharyya - band 3 -  $z > 1.96$

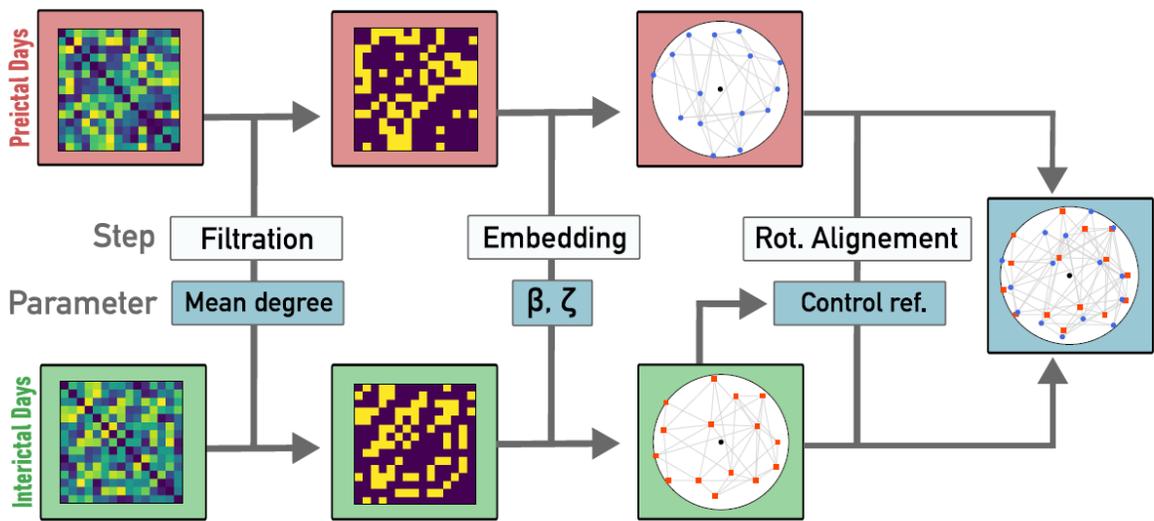




Wasserstein - band 3 -  $z > 1.96$

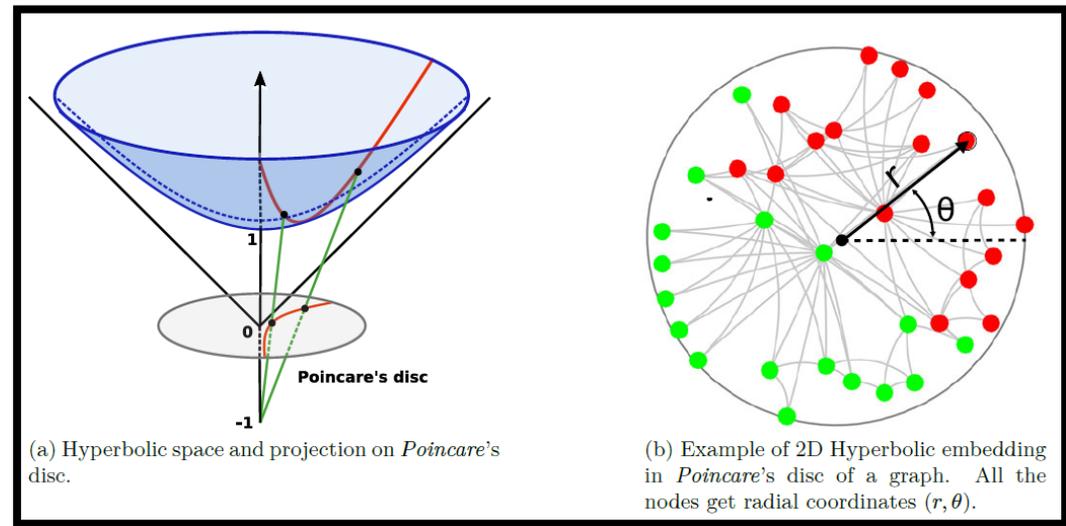


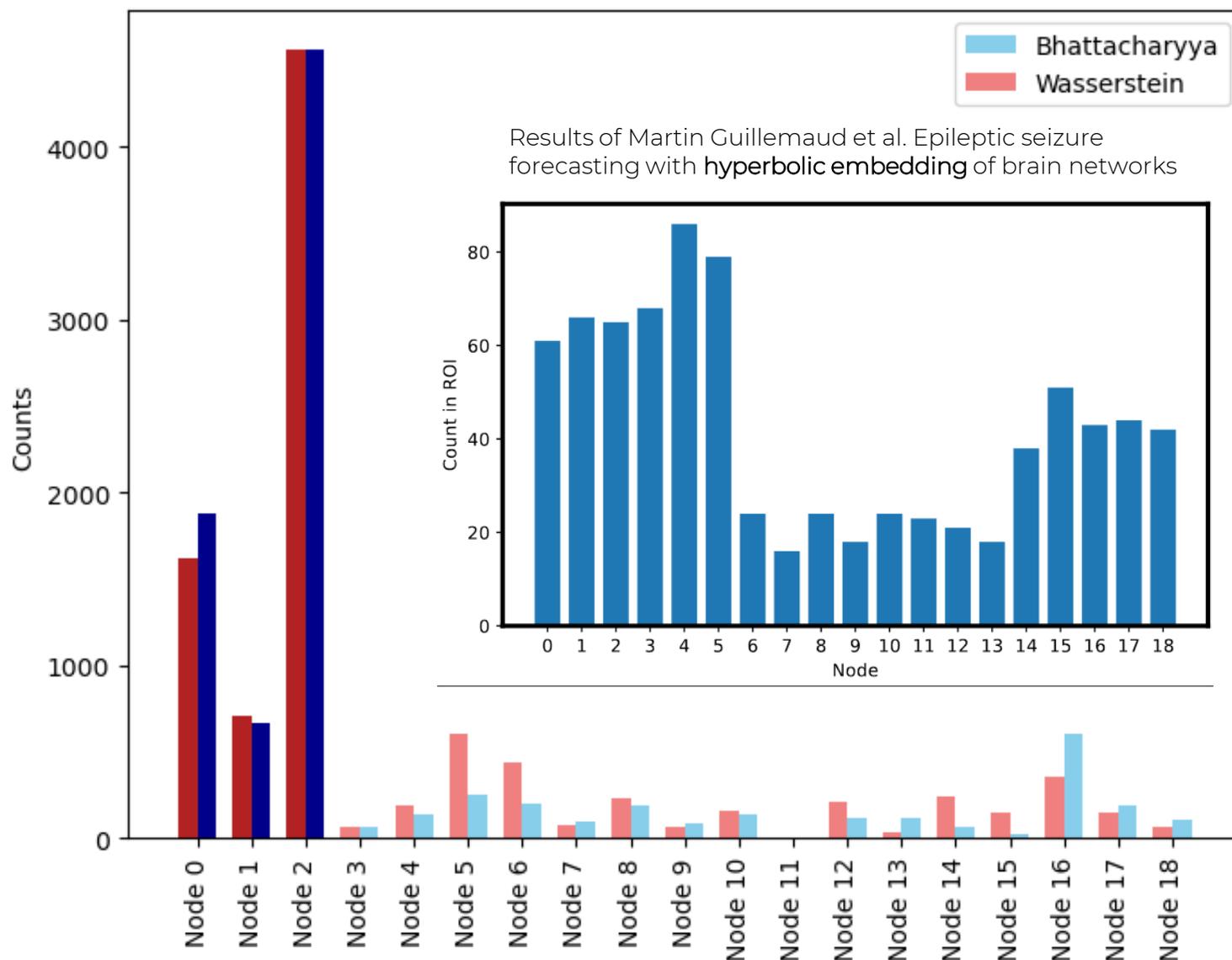
Martin Guillemaud et al. Epileptic seizure forecasting with hyperbolic embedding of brain networks. Article submitted, 2023.



**Figure 1:** Hyperbolic embedding and re-alignment of brain connectivity networks.

Embedding these networks in hyperbolic geometry has high likelihood of identifying pre-seizure brain connectivity patterns and give light under the success prediction of the epilepsy surgery outcome.





## Conclusion

In a first approach, embedding iEEG data in the Euclidean space of diffusion maps allows us to identify local connectivity patterns in the network nodes, highlighting those with potential to differentiate seizure states in epilepsy patients.

## Impact

This is information that is of interest to clinical staff. We can go back to the brain and point out the region of interest where something is happening. We can also correlate if our node corresponds to the center of the seizure event.



Data

Diffusion Map

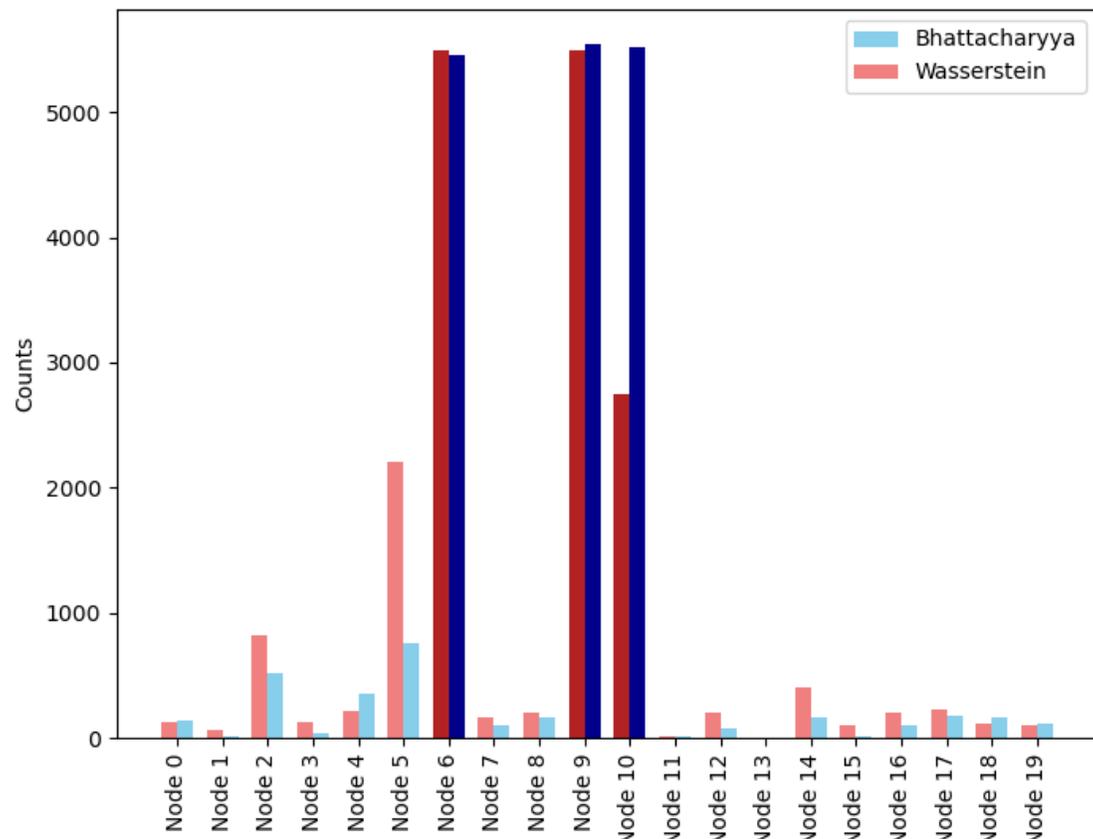
Procrustes

Distances

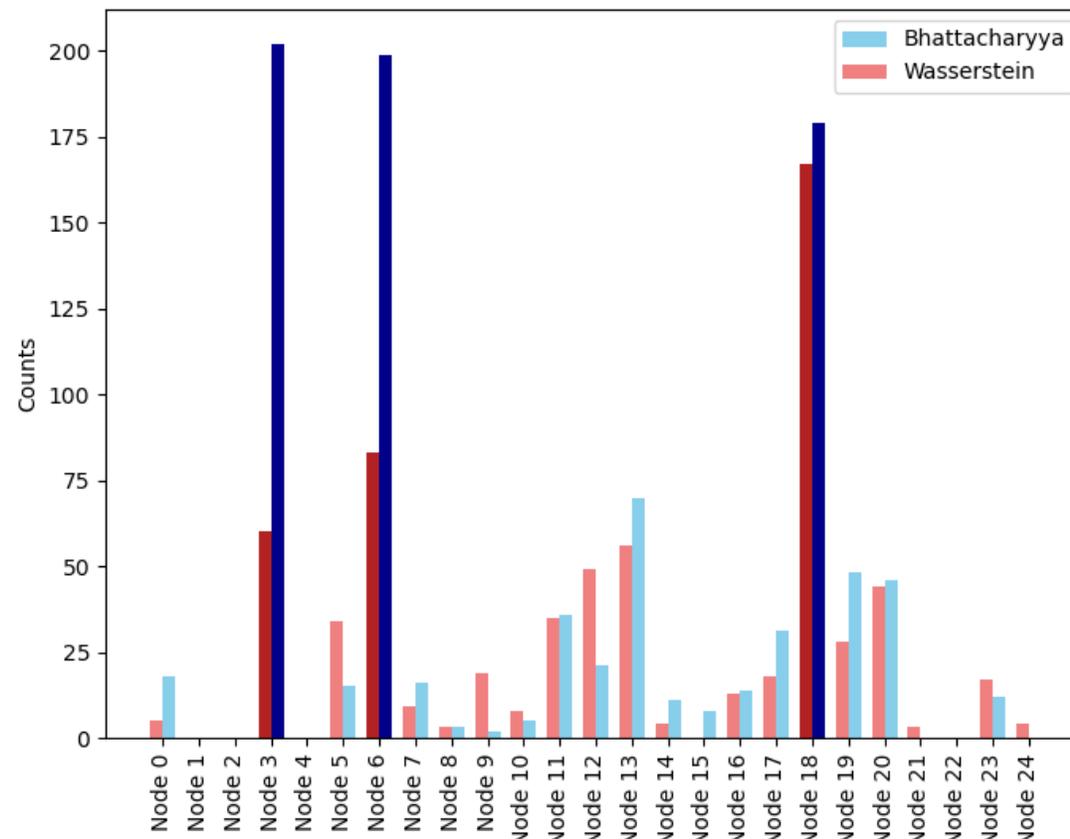
Nodes



band 2 -  $z > 1.96$



band 5 -  $z > 1.96$





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